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Using the Voyage 200 (OS 3.10) in the classroom: surprising results

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ABSTRACT

We teach a variety of math topics (review of College Algebra, Calculus, Differential Equations, applied probability and statistics...) in a Technical Engineering School. The Voyage 200 (or TI 89 Titanium) is mandatory for all new full-time students. We make use of this calculator on a regular basis, for exploring with students, in the classroom, all the classical curriculum in mathematics.

Over the years, using as often this tool as led us to encounter examples where the answers given by the calculator may be unsatisfactory, or strange or even frankly false. On some occasion, this CAS system couldn't resolve a problem which should obviously be easy to solve. Our goal isn't to criticize the Voyage 200 calculator. All CAS system will produce, on occasion, some strange results. This can be explained by choices made in programming the system or by assumptions made to ensure that a result can be obtained. On other occasions, a better understanding of the way this CAS works can help understand results which can be surprising. We will illustrate with examples some of these strange behaviors, trying to give an explanation for why this happens or suggesting some modifications which could eliminate them.

This technology, on every student's desk, is a powerful but inexpensive tool for exploring mathematics. Users will end up, solving more complex problems, doing more math than with the classical approach. But the examples showed here illustrate the fact that students still have to know and understand all the basic math curriculum. The CAS system will not think for them!

1. Introduction

First of all, we have to say that the examples included in this text — some of them were performed live at the conference during our talk — would not have been possible without our continuous use of the device, day after day, in the classroom, with our students. We are proud of this because, even today in 2006, there are still many mathematics teachers who say that they are using technology but, in fact, they rarely do it in front of the classroom. So, the feedback from the students is poor and an opportunity to improve the CAS being used is lost.

We decided to divide these “surprising results” into 3 different — but connected — areas: simplification problems, computation of simple and multiple integrals and lastly, solving equations and systems of equations.

Of course, we know that “what is simple?” is a terrible question to answer for those who are developing computer algebra systems. We also agree that it isn’t easy to decide what a satisfying simplified form for an algebraic expression is. So, what we are doing here is showing some problems or awkward results and we will do our best to collaborate in order to help the developers to solve them.

2. Problems involving simplifications

Sometimes, the TI does not “simplify” enough: for example, putting everything over the same denominator will help for computing successive derivatives of a function involving square roots.

Example 2.1: $x\sqrt{25-x^2} + \sqrt{25-x^2}\sqrt{x^2+24} \rightarrow a(x)$ A memory problems appears if we want to compute the (first!) derivative of this function. We also see, in figure 1a, below, that the TI simplifies the expression for $a(x)$ before trying to compute its derivative:

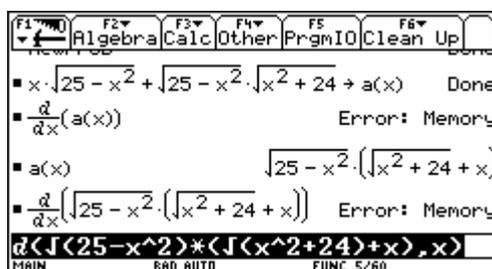


Figure 1a

It is interesting to notice in this case that the problem isn’t with applying the derivative rules but is with the addition of fractional terms containing square roots. In the following screens, we have calculated separately the derivative of each term and asked the TI to sum the two results, which leads again to an “Error: Memory” result. But if we put on a common denominator either one of these two results, the TI will be able to sum the expression. We have tested with other values of expressions similar to the sum of four rational terms involving square roots, like in figure 1c, and have found that the presence of x^2 is important in the square roots and in the numerator of the fourth term in order to reproduce this problem. If you only put the variable x instead of x^2 , in the

numerator of the fourth term, the TI will have no problem computing and simplifying the result

even if you add a fifth term, $\frac{\sqrt{25-x^2}}{\sqrt{x^2+24}}$ for example (see figure 1d).

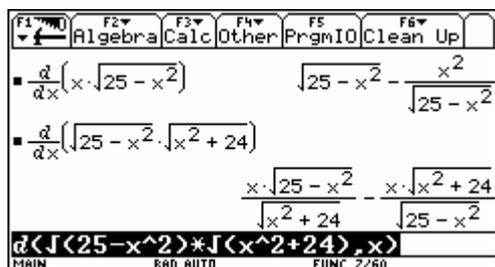


Figure 1b

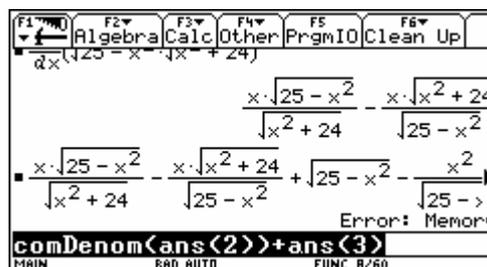


Figure 1c

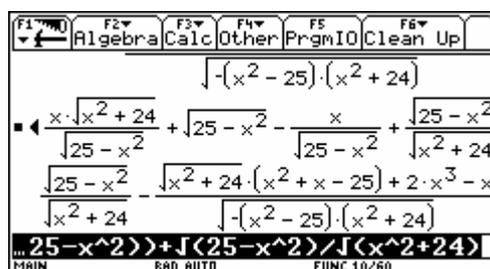


Figure 1d

Example 2.2: if we try to compute the fourth order Taylor polynomial of the expression $f = \sqrt{1+x^3}$, the same « memory error message » appears (Figure 2a). We can see that the TI computes easily the fourth derivative of f if the third one has been put over a same denominator (Figure 2b):

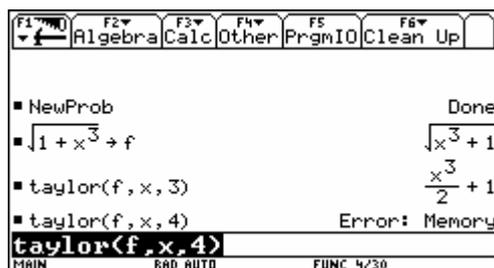


Figure 2a

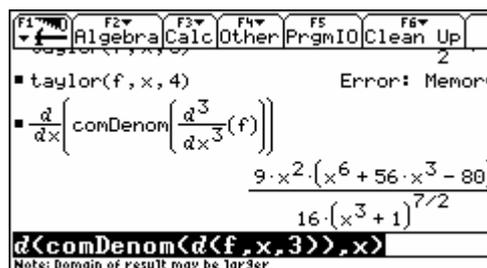


Figure 2b

Being mathematics teachers, we tell our students that if they want to compute some Taylor polynomial, they should, first, try to use a known series — the binomial series in this case — instead of performing all the required derivatives. Or we might suggest using the Taylor command for $\sqrt{1+u}$ and then make the substitution $u = x^3$. It is interesting to notice in this example that the TI wasn't able to compute the fourth derivative without using a common denominator like in figure 2b. In fact, as is shown in figure 2c, the calculator isn't able to compute the derivative of the right side of the expression obtained after differentiating three times.

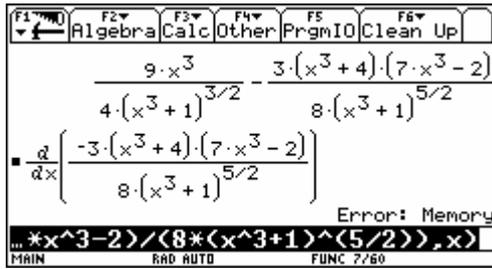


Figure 2c

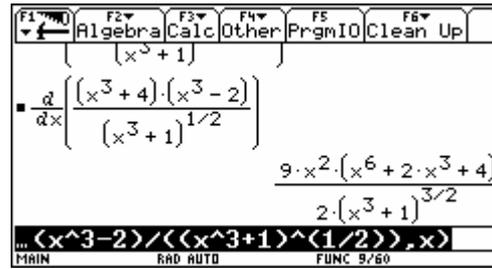


Figure 2d

The calculator even failed to compute the first derivative of the following simpler expression:

$$\frac{(x^3 + 4)(7x^3 - 2)}{\sqrt{x^3 + 1}}$$

which from a standpoint of applying derivative rules is surprising. But if you try without the constant 7 in the numerator, the TI easily calculates the derivative, figure 2d. Troubling...

Example 2.3: sometimes, the device factors out an exponential ... and this seems to be the reason why the computation of a limit fails: for example

$$\lim_{t \rightarrow \infty} -4(2 \cos t + \sin t)e^{-t/2} + 8.$$

Of course, without the additional term « 8 », no problem at all because the TI “knows” the result about the limit of a product of a bounded function and a vanishing function. Figure 3a) below shows how the TI factors out the exponential even if it does not multiply the term “8”. This is why the device seems unable to compute the limit. With no additional term, the computation is done (figure 3b).

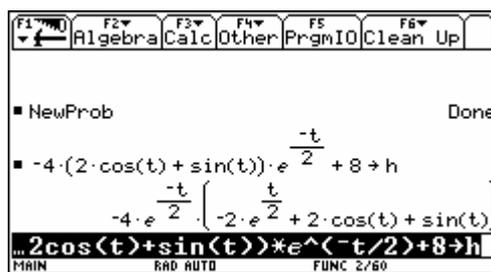


Figure 3a

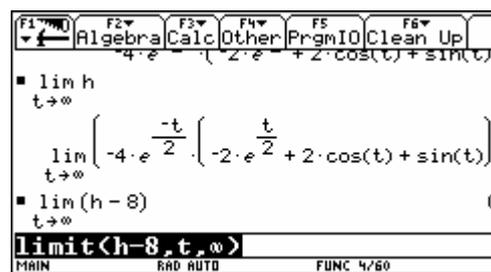


Figure 3b

What can be considered an annoying simplification in figure 3a (we would greatly prefer the TI leaving the expression as is) seems to be a handicap when comes time to perform the limit. It is interesting to notice that if we use e^{-t} instead of $e^{-1/2}$, the calculator doesn't have the problem mentioned above, see figure 3c. Troubling again...

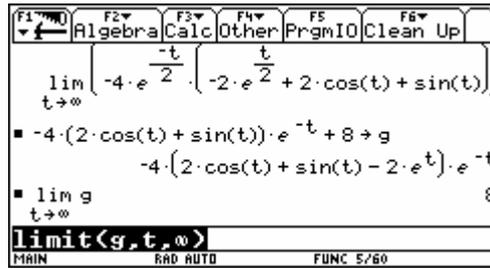


Figure 3c

Example 2.4: two integrals where the TI should expand before computing. Consider the following double integrals:

$$\int_0^2 \int_{x^3-x}^{x^2+x} (x^3 y^4 + xy^2) dy dx; \quad \iint_D (-7x^6 y^6 - 5x^4 y^4) dA, D \text{ the unit disk.}$$

Every student at Ecole de technologie supérieure has a Voyage 200 and will never compute this kind of double integrals by hand! He (she) will use the TI (after spending time on finding the limits of integration, plotting graphs and so on to obtain the first integral and using Green-Riemann formula for the second one). For the first integral, we decided to abort after more than 4 minutes. But, expanding the inner integral yields the results (figure 4a). For the second integral, the same situation occurs (figure 4b):

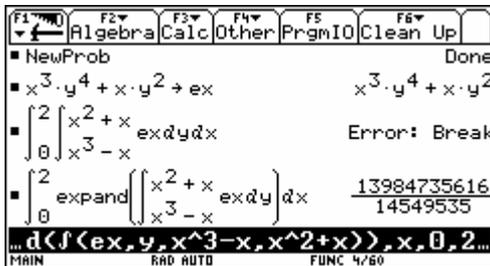


Figure 4a

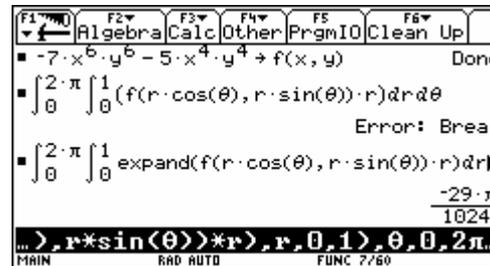


Figure 4b

Figure 4a showed that expanding the inner integral is a good choice. By the way, this is what *Derive* is doing: take a look at the next figure where we have made use of the “show step simplification” feature of *Derive* 6.10:

#1:
$$\int \frac{x^4 \cdot (3x^{14} - 15x^{12} + 30x^{10} - 3x^9 - 45x^8 - 30x^7 - 10x^6 - 15x^5 - 21x^4 - 5x^3 - 15x - 10)}{15} dx$$

$\int a \cdot F(x) dx \rightarrow a \cdot \int F(x) dx$

#2:
$$- \frac{\int x^4 \cdot (3x^{14} - 15x^{12} + 30x^{10} - 3x^9 - 45x^8 - 30x^7 - 10x^6 - 15x^5 - 21x^4 - 5x^3 - 15x - 10) dx}{15}$$

Expand polynomial.

#3:
$$- \frac{\int (3x^{18} - 15x^{16} + 30x^{14} - 3x^{13} - 45x^{12} - 30x^{11} - 10x^{10} - 15x^9 - 21x^8 - 5x^7 - 15x^5 - 10x^4) dx}{15}$$

Figure 5

It would be a good idea if, like DERIVE, the TI would have, as part of its computing procedure, an “Expand” command to try to solve integrals when clearly it is failing to produce an adequate result.

Example 2.5: this example has also to do with expansion, but expansion of numbers. Many of our students were not able to see that $\sqrt{10-4\sqrt{6}} = \sqrt{6}-2$. The TI does. The reader will probably want to know why this specific number is being under consideration. Our students had to solve the polynomial equation $d^4 - 20d^2 + 4 = 0$; this number is one of the 4 roots of this equation. We think that the TI should simplify $-2(2\sqrt{6}-5)$ into $10-4\sqrt{6}$.

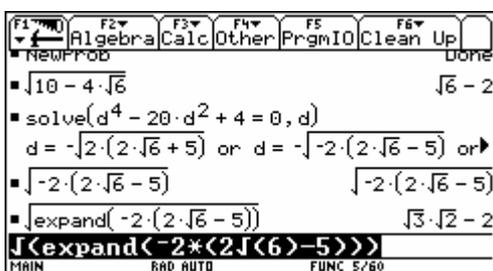


Figure 6a

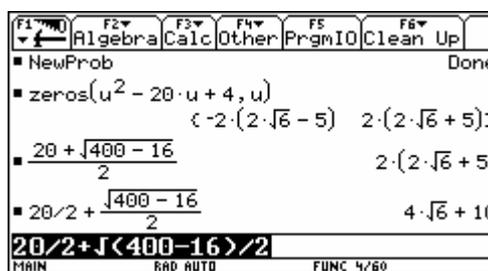


Figure 6b

Let us make it clear, here. We do not want the TI to implement sophisticated nested radicals simplifications but to be more aggressive with the way numbers and expressions are written. Like *Derive*, the TI has already implemented such simplifications: by using *Derive*, we can understand why $\sqrt{10-4\sqrt{6}} = \sqrt{6}-2$:

If $x > y > 0$,

$$\sqrt{(x-y)} \rightarrow \sqrt{\frac{x + \sqrt{(x^2 - y^2)}}{2}} - \sqrt{\frac{x - \sqrt{(x^2 - y^2)}}{2}}$$

Figure 7

The problem in this instance has more to do with the way the TI calculates the roots of a quadratic equation. To illustrate this let's consider the original problem above and its auxiliary equation $u^2 - 20u + 4 = 0$. The TI solves this and gives answers not in the form $x - y$ which then prevents it (when taking the radical) from using the transformation mentioned above to give a simpler form for the four roots, see figure 6b.

Even doing calculations manually with the quadratic formulas yields the same problem when the TI simplifies the expression $\frac{20 \pm \sqrt{400-16}}{2}$; we would prefer the TI giving us answers in the form $10 \pm 4\sqrt{6}$. We obtain this last form if calculations were made using $\frac{20}{2} \pm \frac{\sqrt{400-16}}{2}$. These numbers should be given, if possible, in the form $a + b\sqrt{c}$, where $a, b \in \mathbb{Z}$ and $c \in \mathbb{N}$. This can be obtained on the TI by applying the following sequence of commands on numbers like $-2(2\sqrt{6}-5)$: 1° expand() 2° ans(1) then enter.

3. Problems with integrals

We can divide the next examples into 2 cases : first, the symbolic integrator encounters certain problems — sometimes related to some inner simplifications or absent built-in functions — and, second, we give a good example of a wrong simplification that leads to an inexact numerical value of a triple integral.

Example 3.1: we know that $\sqrt{\frac{1}{b}} \neq \frac{1}{\sqrt{b}}$ unless b is positive. So, we are not surprised to note that

the TI does not give a closed form for the integral $\int \sqrt{\frac{1}{x^2-1}} dx$ and, consequently, gives a floating

point answer for $\int_1^2 \sqrt{\frac{1}{x^2-1}} dx$.

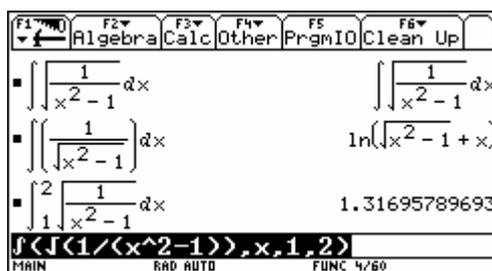


Figure 8

For the definite integral, it would be nice if, “knowing” that the limits of integration are greater than one, then $\sqrt{\frac{1}{x^2-1}} = \frac{1}{\sqrt{x^2-1}}$ and $\int_1^2 \frac{1}{\sqrt{x^2-1}} dx = \ln(\sqrt{3}+2) \approx 1.31696$. But we still think that

this is a good opportunity for the math teacher to play an important role: when the CAS is asked for a definite integral, it first computes the antiderivative and, then, evaluate at both limits. By the way, the reader can take a look at the answers given by popular CAS systems (*Derive*, *Maple* and

Mathematica). They get a closed form for $\int \sqrt{\frac{1}{x^2-1}} dx$, but this requires complex numbers (for

Derive) or an evident choice of appropriate domain for the other CAS. See the next figure. A final comment on this example: before using the symbolic TI calculator, many teachers were not paying too much attention to properties of log. For example, they were writing on the blackboard formulas like

$$\ln(ab) = \ln(a) + \ln(b)$$

forgetting to write down the additional condition (a and/or $b > 0$). The fact that a CAS won't simplify $\ln(ab) - \ln(a) - \ln(b)$ to 0 without any knowledge about a and b is very useful in the teaching of mathematics.

$$\int \sqrt{\frac{1}{x^2 - 1}} dx = \text{L} \cdot \text{ASIN}(x)$$

(Derive 6.10)

> **int(sqrt(1/(x^2-1)),x);**

$$\sqrt{\frac{1}{x^2 - 1}} \sqrt{x^2 - 1} \ln(x + \sqrt{x^2 - 1})$$

(Maple 9.5)

$$\int \sqrt{\frac{1}{x^2 - 1}} dx =$$

$$\sqrt{\frac{1}{x^2 - 1}} \sqrt{x^2 - 1} \log(x + \sqrt{x^2 - 1})$$

(Mathematica from <http://integrals.wolfram.com/index.jsp>)

Figure 9

Example 3.2: because the Error function is not a built-in function in the TI, we can understand why it does not get a closed form for the following integral: $\int e^{x^2} (2x^2 + 1) dx$. But we have a problem with the following: $\int e^{x^3+x} (3x^2 + 1) dx$. Integration by substitution is (probably) one of the most important methods of integration. Here, we have the form $\int e^{f(x)} f'(x) dx$. We just can't understand why the TI expands the integrand instead of verifying the basic substitution form:

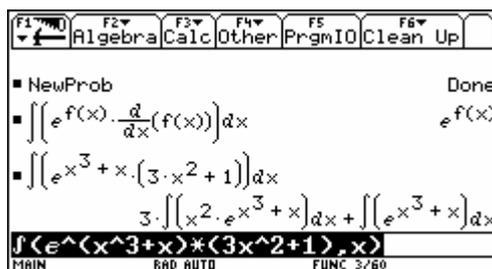


Figure 10

Example 3.3: when our students, in a multiple variable calculus course, have to use cylindrical coordinates for the computation of a given triple integral, they also use the given rectangular coordinates in order to check their answer — because the TI is doing the calculations! — If both answers are the same (at least, numerically), they are confident that their triple integral was set correctly! We found an interesting example. Suppose you want to compute the volume of a solid that is bounded below by the plane $z = 0$, above by the cone $z = \sqrt{x^2 + y^2}$ and on the sides by the cylinder $x^2 + y^2 = 2y$. You will have to compute the following triple integral:

$$V = \int_{-1-\sqrt{1-x^2}}^{1+\sqrt{1-x^2}} \int_{-\sqrt{x^2+y^2}}^{\sqrt{x^2+y^2}} \int_0^{\sqrt{x^2+y^2}} dz dy dx$$

Using cylindrical coordinates, we easily get the exact answer: $\frac{32}{9} \approx 3.55556$. When the students compared the answer obtained using rectangular coordinates, they started to be convinced of the importance of cylindrical coordinates!

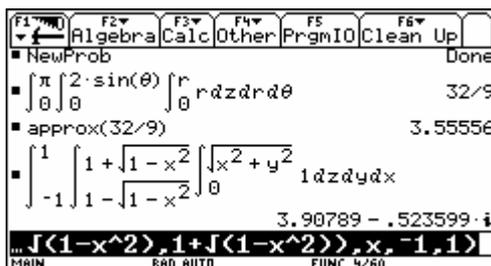


Figure 11a

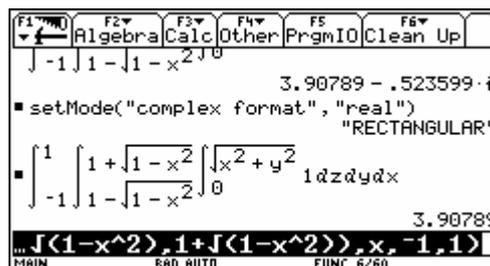


Figure 11b

But some of them used the fact that the volume must be equal to $2 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} \int_0^{\sqrt{x^2+y^2}} dz dy dx$. With this new triple integral, the TI makes no error. In fact, the problem seems to be caused by the following fact: when the inner double integral has been computed, the integrand, as a function of x ,

is not an even function — in fact it is not defined for negative x —, so the device could not use the

shortcut $\int_{-a}^a f(x) dx = 2 \int_0^a f(x) dx$, for an even function f .

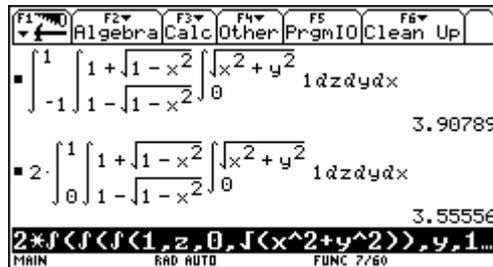


Figure 12

And here is the mistake made by the TI (this is what we think!): when evaluating the integral

$$\int_{1-\sqrt{1-x^2}}^{1+\sqrt{1-x^2}} \sqrt{x^2 + y^2} dy,$$

the device correctly used the antiderivative

$$\frac{x^2 \ln(y + \sqrt{x^2 + y^2})}{2} + \frac{y\sqrt{x^2 + y^2}}{2}.$$

Let us substitute for y , the lower limit of integration, namely $1 - \sqrt{1 - x^2}$. The square root inside

the log becomes $\sqrt{x^2 + (1 - \sqrt{1 - x^2})^2} = \sqrt{2 - 2\sqrt{1 - x^2}}$. Then, using the rule shown in figure 7, we

can write, for $-1 < x < 1$, $\sqrt{2 - 2\sqrt{1 - x^2}} = \sqrt{|x| + 1} - \sqrt{1 - |x|}$. Which is equal to $\sqrt{x + 1} - \sqrt{1 - x}$

if and only if $0 \leq x \leq 1$. But the TI wrongly simplified $\sqrt{2 - 2\sqrt{1 - x^2}}$ into $\sqrt{x + 1} - \sqrt{1 - x}$ as figure 13 shows:

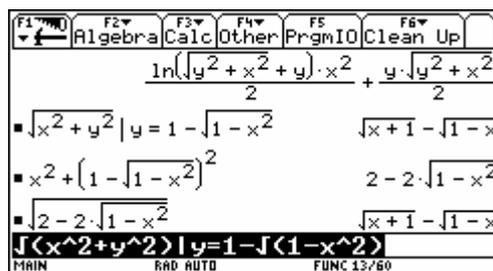


Figure 13

4. Problems with the solving facilities

Here we move to a different area.

Example 4.1: a very funny example! The following equation $2 \cos \theta + \sin \theta = 1$ with $0 \leq \theta \leq \pi$ has the unique solution $\pi/2$. The TI finds it ... but take a look at the next figure:

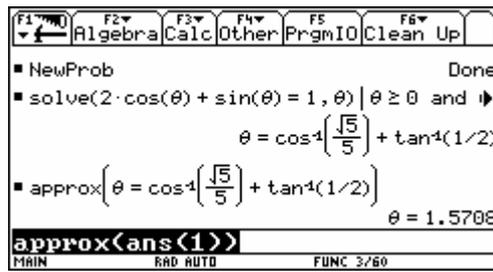


Figure 14

In fact, we decided to exhibit this example because we think that this is part of « symbolic magic ». We suspect the TI to have used some trigonometry, probably “tCollect”, obtaining

$$2 \cos \theta + \sin \theta = \sqrt{5} \cos(\theta - \tan^{-1}(1/2)).$$

Now, how do we « simplify » $\cos^{-1}\left(\frac{\sqrt{5}}{5}\right) + \tan^{-1}\left(\frac{1}{2}\right)$ into $\frac{\pi}{2}$? Here is an answer, given by

Derive:

$$\#1: \quad \text{ACOS}\left(\frac{\sqrt{5}}{5}\right) + \text{ATAN}\left(\frac{1}{2}\right)$$

$$\text{ACOS}(z) \rightarrow \frac{\pi}{2} - \text{ASIN}(z)$$

$$\#2: \quad \frac{\pi}{2} - \text{ASIN}\left(\frac{\sqrt{5}}{5}\right) + \text{ATAN}\left(\frac{1}{2}\right)$$

$$\text{ASIN}(z) \rightarrow \text{ATAN}\left(\frac{z}{\sqrt{1-z^2}}\right)$$

$$\#3: \quad \frac{\pi}{2} - \text{ATAN}\left(\frac{1}{2}\right) + \text{ATAN}\left(\frac{1}{2}\right)$$

Figure 15

Example 4.2 : we are very impressed with the way the TI solves trigonometric equations, giving results we are looking for when the domain of the variable is specified (this feature was not available some years ago). For example:

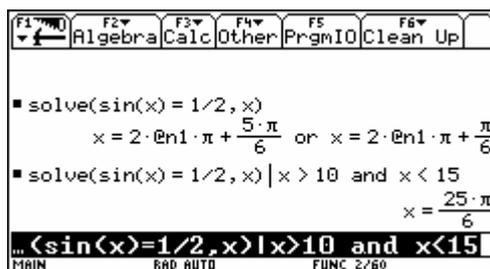


Figure 16

But this feature seems to work with linear argument only : an equation like $\sin(x^2)=0$ is correctly solved but when we try to solve for x between -3 and 3 , we only get the positive solutions and the symbol “@n3” should not be part of the answer:

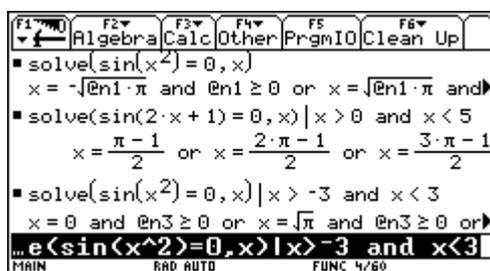


Figure 17

Example 4.3: here, some lacking — but not essential! — implementation is probably responsible for what happens. First, consider the system (“syst1”) of 2 equations in 2 unknowns:

$$\text{syst1: } \begin{cases} \sin y = 0 \\ x \cos y = 0 \end{cases}$$

The solutions are $(0, k\pi)$ with $k \in \mathbb{Z}$. This answer should be the only one but another strange one appears (why is the TI considering that $(-1)^n$ can be equal to 0?):

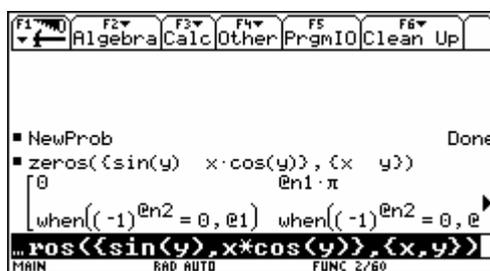


Figure 18

Second, consider the system $\text{syst2: } \begin{cases} an2^{n-1} = 3 \\ an4^{n-1} = 24 \end{cases}$. This system comes from a calculus problem

where the students had to find a power function of the form $f(x) = ax^n$ satisfying $f'(2) = 3$

and $f'(4) = 24$. The TI gives a (correct) floating point answer for the (exact) answer: $a = 3/32 \approx 0.09375$ and $n = 4$. But, asking to solve for only one variable, namely a , we get the exact answer! And asking to solve for only the variable n , the 2 equations are returned (because the Lambert W function — inverse function of xe^x — is not a built-in function in the TI). This confused the students and this is, again, a good opportunity to tell the students that some pencil and papers techniques are still useful for solving equations. It is also a good occasion to discuss with students the correct syntax for these commands, showing the importance of indicating the list of variables instead of only one (a bad habit they pick up solving system of linear equations).

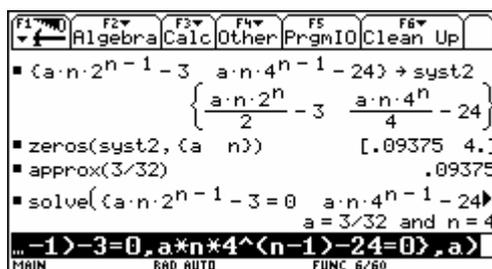


Figure 19

Example 4.4: A serious bug. In OS 2.09, many systems of equations were solved easily when a simple **change of name** for one of the unknown was made! In fact, **it seems that we should not choose a Greek letter**. We thought that this problem was resolved with OS 3.10 but here are 2 more examples. They are not linear systems — not even polynomial systems — but the problem is serious in the sense that the TI should not be sensitive to a particular choice of names for the unknowns: solving $2x + 4 = 0$ for x or solving $2\beta + 4 = 0$ for β should give the same answer. The next 2 examples show that it is not always the case for a system of equations.

$$\text{andy11:} \begin{cases} -tac \cos \alpha + tab \cos \beta = 0 \\ tac \sin \alpha + tab \sin \beta - w1 = 0 \\ -tab \cos \beta + tbd \cos \theta = 0 \\ -tab \sin \beta + tbd \sin \theta - w2 = 0 \end{cases} \quad \text{with } w1 = 100, w2 = 80, \alpha = \theta = 45^\circ .$$

The unknowns are tab , tac , tbd and β . Using « zeros » yields the empty list with the message « warning: solve may specify more zeros ». Using solve with “and” between each equation, we get the following answer (see figure 20a):

$$tab = 90.5539 \text{ and } tac = 127.279 \text{ and } tbd = 127.279 \text{ and } \beta = 0.110657$$

Using “solve” with {}, we get the following — equivalent but strange! — answer (see figure 20b):

$$tab = 90.5539 \text{ and } tac = -128.062(\sin \beta - 1.10432) \text{ and } tbd = 127.279 \text{ and } \beta = 0.110657$$

Trying again with the “zeros” function and changing the letter β by the letter y we get the answer! See figure 20b. So, the “zeros” function returns the empty list when the letter β is used and a correct answer when the letter y is chosen.

Here is another such bug for a system. Consider the system “andy12”:

$$\text{andy12:} \begin{cases} -T + us * Na + ma * g * \sin \theta = 0 \\ Na - ma * g * \cos \theta = 0 \\ -T - us * Na + mb * g * \sin \theta = 0 \\ Nb - Na - mb * g * \cos \theta = 0 \end{cases} \quad (ma, mb, g, \text{ and } us \text{ are 4 positive parameters})$$

Unknowns are θ, T, Na and Nb . The « zeros » function gives the empty list but, when we use the letter a instead of θ , a symbolic answer is obtained (see figure 20c).

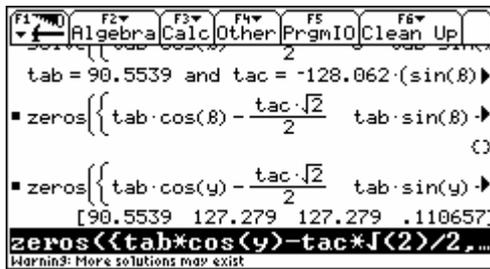


Figure 20a

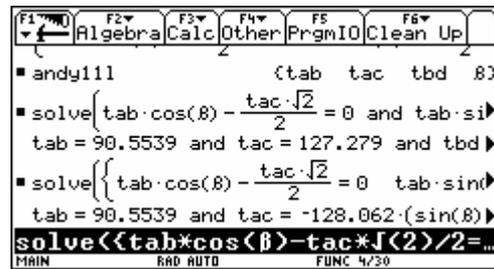


Figure 20b

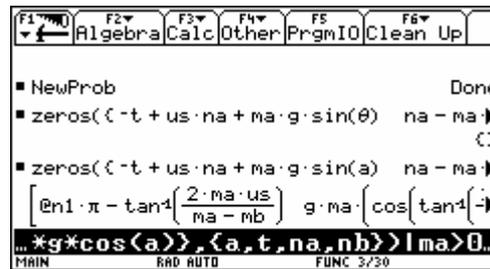


Figure 20c

Example 4.5: a troubling example! Our colleague, Chantal Trottier had to grade a question in an exam where students had to use Laplace Transforms to solve a classic spring-mass differential equation with an external force of $6\cos(3t)$. The general equation for this harmonic motion is

$$m \frac{d^2 x}{dt^2} + b \frac{dx}{dt} + kx = f(t),$$

where the constants m, b and k represent the mass, the damping factor and the spring constant. The external force is $f(t)$. For this problem, students had to resolve using the following values: $m=3, b=6$ and $k=15$, with initial position $x(0)=0$ and initial velocity of $v(0) = x'(0) = 3$. Taking Laplace transform of these equations leads us to solve for X the

following equation: $m(s^2X - sx(0) - x'(0)) + b(sX - x(0)) + kX = \frac{6s}{s^2 + 9}$ or in this case

$3(s^2X - 3) + 6(sX) + 15X = \frac{6s}{s^2 + 9}$. Some Students make mistakes when substituting the

assigned values in the general equation so she created a function on her calculator to verify what students should obtain before taking the inverse Laplace transform of the solution. Using x for X and replacing in the general equation $x(0)$ and $x'(0)$ by x_0 and v_0 and storing in the function $chantal(m,b,k,x_0,v_0)$ the command to solve the general equations and then asking the TI to evaluate the expression for $chantal(3,6,15,0,3)$, you get the desired response, figure 21a.

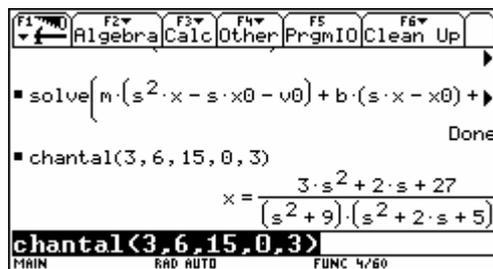


Figure 21a

The student is now ready to apply inverse Laplace techniques for finding the response in the time domain. But if you try the same function with a damping constant of 5 instead of 6, you get the surprising following result, figure 21b. The TI seems unable to solve for x this equation even if it is linear in this variable. Even worst, if you take this result and ask the TI to solve it for x , it does the right thing. As if, asking in a user-defined function to solve an equation isn't equivalent to solving it in the Home environment.

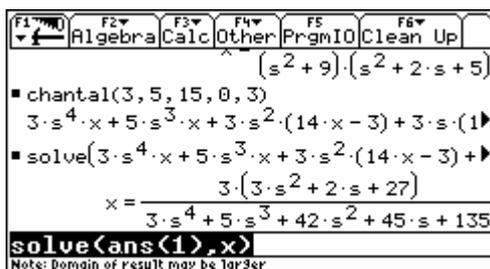


Figure 21b

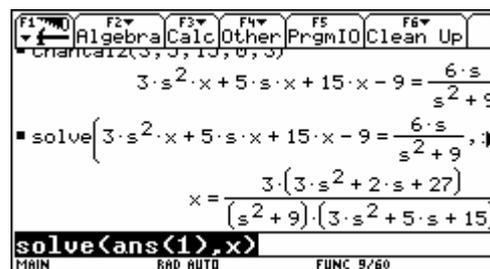


Figure 21c

If we create a new function to obtain only the desired equation to be solved, let's name it $chantal2(m,b,k,x_0,v_0)$ and then solve the result for x in the Home environment, we don't encounter any problem but we do obtain two different form of the desired result, see figures above. The algebraic form obtained when applying the function $chantal(3,5,15,0,3)$, see figure 21b, where the TI refuses to solve the equation is the same as the one obtained when the calculator tries

to solve the general case $chantal(m,b,k,x0,v0)$, see figure 21d. Again, if we ask in the Home environment to solve the general equation, we do get (after a long calculation) the desired result, see figure 21e.

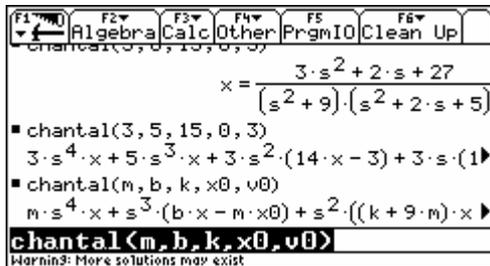


Figure 21d

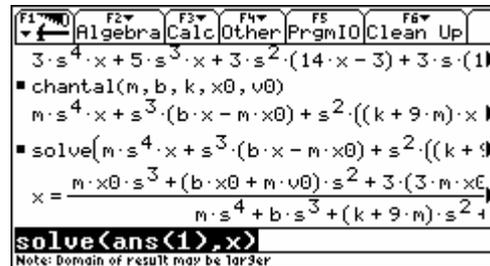


Figure 21e

This problem occurs with some combination of numbers. For example, $chantal(2,4,16,0,2)$ or $chantal(2,3,6,0,2)$ will produce this problem but $chantal(2,4,9,0,2)$ or $chantal(9,6,3,0,3)$ will work fine. And when the problem occurs, you can always solve in the Home environment but you will not get a factorized denominator as you would with the procedure in figure 21c. We were not able to find a rule for cases that work versus combination of number who don't work. Now for the most troubling part: after playing around this problem and trying many combination of number we found out that $chantal(3,5,k,0,3)$ works fine but replacing k with the value 15 creates the problem. This means the TI is able to evaluate a more general case! Strange...

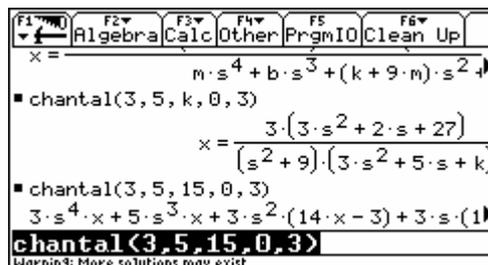


Figure 21f

5. Conclusion

Although you see in this paper many examples of strange or deficient behavior of the Voyage 200 calculator, we must emphasize that these cases were rounded up by many users and teachers over many years of constant usage in math teaching (and we are glad that we only found the few examples compiled here). All CAS system will have, on occasion, these strange or odd behavior and this is to be expected. We do understand that choices have to be made and that solving a problem can create another one in a different context. Our contribution here is to help developers

of this symbolic calculator to achieve an even more powerful and stable CAS system for the benefit of all of its users. We wanted also to underline the fact that the Voyage 200 is only a tool to better explore some mathematical aspects; we still have to teach the classic topics in mathematics. Students have to understand how the Voyage 200 gets its results and be able to verify if the results are valid.

We would like to thank the following colleagues for letting us use some of their examples: Robert Michaud, Geneviève Savard, André St-Amand and Chantal Trottier. Being enthusiastic users of the Voyage 200, we would be happy to help TI in trying to solve some of the problems showed in this text!