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**Teaching Mathematics to Engineering Students:
To Use or Not To Use TI-Nspire CAS**

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We have been using the Voyage 200 to teach a variety of math topics in a Technical Engineering School (ETS in Montreal, Canada). We have to admit that the Voyage 200 is doing a very good job but, for some problems, a much faster processor would be helpful and would allow more powerful graphic options. This fast processor is exactly what we now have in the TI-Nspire CAS. So, when you need to solve some “heavy problems”, say complicated equations, polynomial systems of equations, or when you want to define special functions using a definite integral, the Voyage 200 processor cannot compete with the TI-Nspire CAS calculator. The talk will give some examples of this, using Nspire CAS and comparing with Voyage 200 results.

Since we are teaching mathematics to future engineers, we also often need implicit 2D plots, 3D plots and differential equations plotting (along with RK and Euler numerical methods). We have all of this in the Voyage 200 but it can take a lot of time to get some results or see graphs on the screen and, in 3D plotting, the results are limited mainly due to a lack of processor speed. Adding these features to Nspire CAS — and color for the PC version — would be important and, as far as we are concerned, this is a must. If this is not done, 3D plots will continue to be done, by our colleagues and us, using Derive or Maple software. In order to make the move to Nspire CAS software, we absolutely need a much more “university level package”.

1. Introduction

We are very impressed by the Nspire processor but Nspire is still missing “engineering features”. So, let us give examples of both in the next section. Of course, the examples come from our own experience at ETS, where the students are engineering students. So, single and multiple variable calculus, differential equations and statistics will be the subjects among which we will choose these examples.

2. A very fast processor that should also be used at university level

The following examples show clearly a difference of speed between Voyage 200 processor and Nspire CAS processor (handheld and software). We do hope that TI will add some important features to Nspire: implicit 2D plotting, differential equation plotting and 3D plotting. We will illustrate this with our examples.

Example 2.1: plotting a function defined by a definite integral. Special functions like the Error function, the Sine integral function, Fresnel_sine function are not implemented in Voyage 200, neither in Nspire CAS. This is not a problem because this represents a very nice opportunity, in single variable calculus, to illustrate the importance of the fundamental theorem of calculus:

if f is a continuous function over an interval $[a, b]$, then the function g , defined by

$$g(x) = \int_a^x f(t) dt \quad (a \leq x \leq b),$$

is an antiderivative of f : $g'(x) = f(x) \quad (a < x < b)$. When we want to look at this function g , a graph is a good choice... if we get the graph quickly!

Define the function $g(x) = \int_0^x \sin(t^2) dt$ and plot it in the window: $-3 < x < 3, -1 < y < 1$. You would see

that Voyage 200 calculator needs more than 12 minutes, while less than 2 minutes are required for the Nspire CAS handheld and a few seconds with the software.

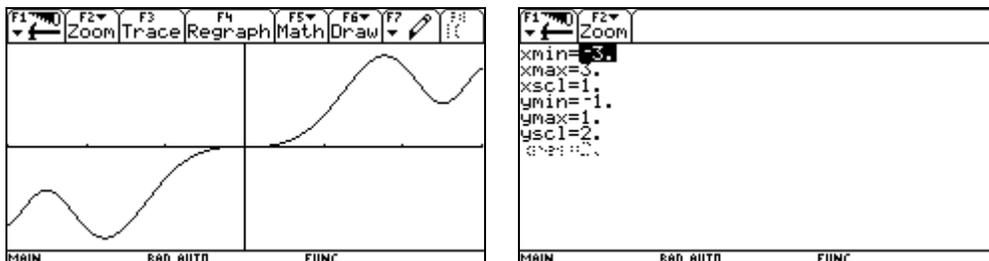


Figure 1

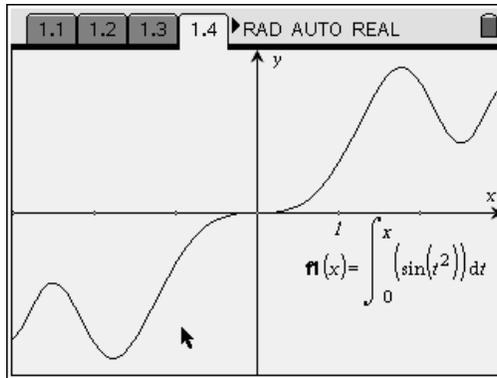


Figure 2

Example 2.2: solving polynomial systems. When we want to find and analyze the critical points of a 2 variables function $f = f(x, y)$, we can plot level curves or plot the curves defined by the system $\nabla f = [0, 0]$ where $\nabla f = \left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \right]$ is the gradient vector. For the Voyage 200, implicit 2D plot is available in the 3D plot window — only one curve per window —, so level curves are a better choice, but plotting is very slow. It would have been nice to see what Nspire can do, because the processor is fast ... but, for the 3D and the implicit 2D, we will have to wait. Of course, Nspire and Voyage 200 are using Groebner basis, so all solutions should be obtained if we use the “solve” command or the “zeros” command. Let us illustrate this with $f = x^4 - xy^2 + 3y^3 - 5y$. We prefer to get the critical points in a list, so the “zeros” function will be used. Voyage 200 takes about 58 seconds to find the 2 critical points while Nspire CAS handheld needs only 2 seconds (Voyage 200 takes only 15 seconds with \blacklozenge Enter but we should avoid this because, sometimes, wrong results are obtained: see the next example).

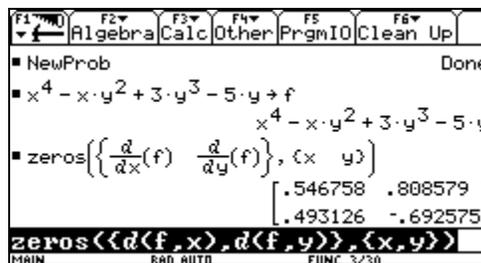


Figure 3

In order to find out the nature of these 2 critical points, we can do further investigations, using namely multiple variable calculus functions like the Hessian matrix (we won't do this, here: but it can be shown that the critical point in the first quadrant is a minimum while the one in the fourth quadrant is a saddle point). Let us conclude this example, by taking a look at some *Derive* screens (figure 4): the 2 critical points along with the curves $\frac{\partial f}{\partial x} = 0$, $\frac{\partial f}{\partial y} = 0$, the 2 critical points with some level curves and, finally, the 2 critical points (in the space) with the surface defined by $z = f(x, y)$.

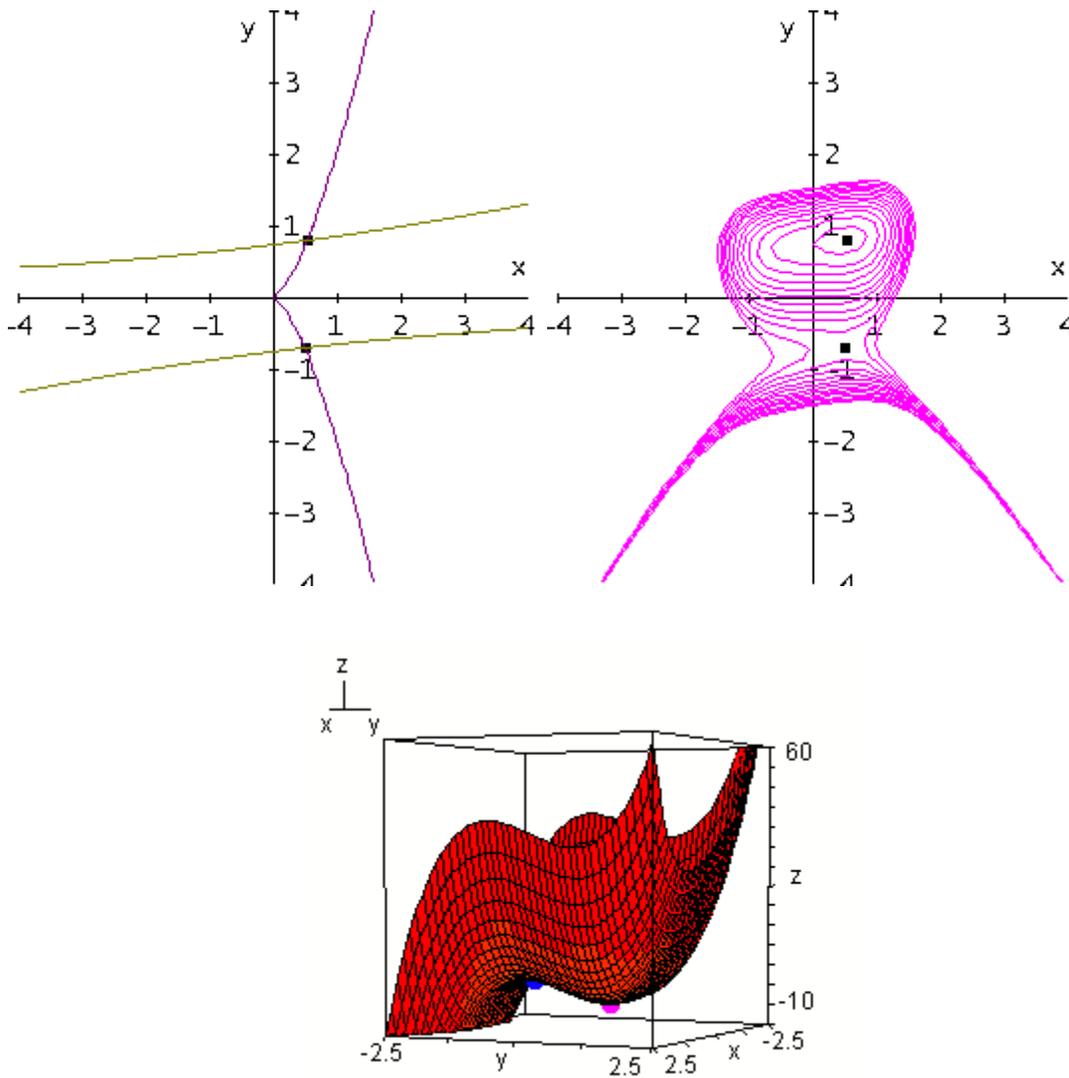


Figure 4

Example 2.3: a faster processor, like the one found in Nspire handheld, can play an important role in classrooms at university level. At our university, the Voyage 200 calculator is used for some “heavy” problems and sometimes students don’t want to wait too long to get their answers ... so, in AUTO mode, they will do ♦ENTER instead of ENTER. The following example shows that this can lead to wrong answers, caused by the solver in this case. Suppose you want to find the maximum and minimum volume of a closed rectangular box having a total surface of 1500 cm² and whose sum of the sides is 200 cm. Using Lagrange multipliers techniques, we need to find the zeros of the following 5 expressions:

$$\begin{cases} yz - \lambda(2y + 2z) - 4\mu \\ xz - \lambda(2x + 2z) - 4\mu \\ xy - \lambda(2x + 2y) - 4\mu \\ 2xy + 2xz + 2yz - 1500 \\ 4x + 4y + 4z - 200 \end{cases}$$

Voyage 200 takes about 100 sec to find (exactly) a 6 × 5 matrix (with entries containing square roots) while Nspire CAS handheld takes about 4 sec! This matrix approximates to the one showed in figure 5:

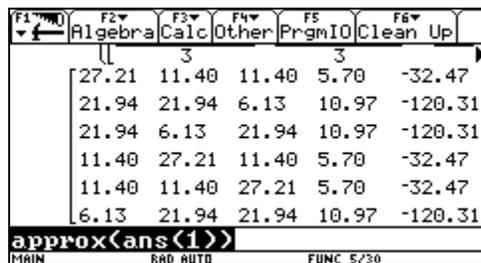


Figure 5

One can check that every solution satisfy the 5 equations. We see, in fact, that there are only 2 different sizes for the box: 27.21 × 11.40 × 11.40 for a volume of 3533.54 cm³; or 21.94 × 6.13 × 21.94 for a volume of 2947.94 cm³. Both Voyage 200 and Nspire CAS give wrong results when we use ♦ENTER (ctrl enter) when solving this system of equations using the “zeros()” command. Look at the following result (figure 6):

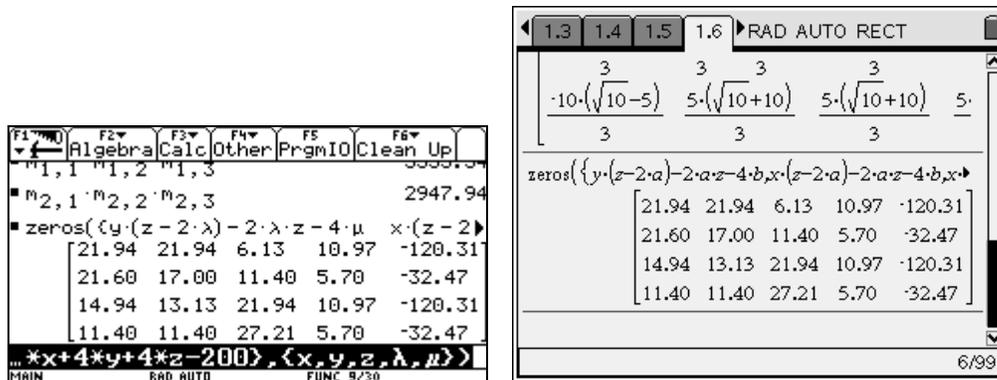


Figure 6

In fact, the “solutions” $x = 21.60$, $y = 17$ and $z = 11.40$ satisfy the second constraint but not the first. The same situation occurs with 14.94 , 13.13 and 21.94 .

Example 2.4: a “dsolve” command without differential equations plotting is not complete! Consider the following example. A body of mass $m = 0.2$ kg is thrown upward from the top of a building of height $h = 150$ m, at an initial velocity of $v_0 = 40$ m/s . Let’s assume that air resistance force, F_R , is proportional to the square of the velocity, so $F_R = \pm kv^2$ where k is a positive constant (consider that the numerical value of k is $1/100$). After how many seconds will the body touch ground? To answer this question, we must solve 2 differential equations (one for upward motion and one for the downward motion). Choosing the positive direction to be upward, we have:

$$m \frac{dv}{dt} = -mg - kv^2, v(0) = v_0 \quad (\text{Upward motion})$$

$$m \frac{dv}{dt} = -mg + kv^2, v(0) = 0 \quad (\text{Downward motion})$$

We can combine these 2 differential equations in a single one if we write

$$m \frac{dv}{dt} = -mg - k|v|v, v(0) = v_0$$

But the absolute value makes the symbolic solving difficult. This is where the RK method of the Voyage 200 becomes very interesting: setting $y = \frac{dv}{dt}$, we are able to predict that the body will reach a maximum height of about 172 m and take 15 sec to hit the ground, without solving anything! See figure 8b (value of g is taken to be 9.81). We use RK as the solution method and no slope field, no direction field (FLDOFF). We type the system of ODEs (figure 7a and 7b):

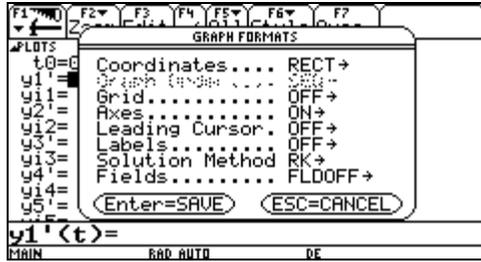


Figure 7a

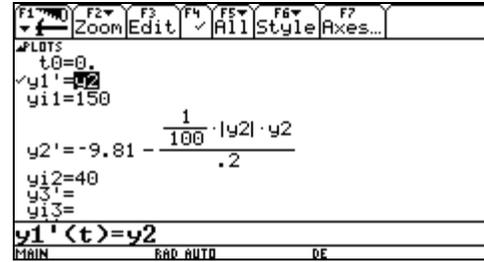


Figure 7b

We set the windows parameters and plot the graph of the position only: figure 8a and 8b:

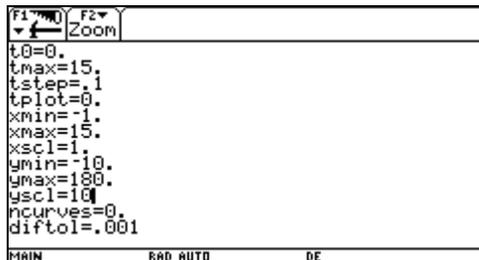


Figure 8a

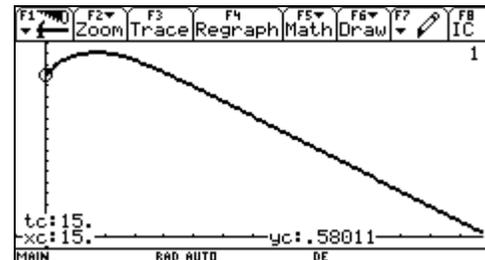


Figure 8b

We know that a “differential graphing mode” will be soon available on Nspire CAS. As the last example shows, this is a must in engineering studies.

Example 2.5: Nspire CAS calculator window extends the Home screen of Voyage 200. In fact, Statistics and Financial Mathematics are no longer “Flash Applications”. The Algebra menu and the Calculus menu have more features — and complete names of the commands are now shown —, see figure 9:

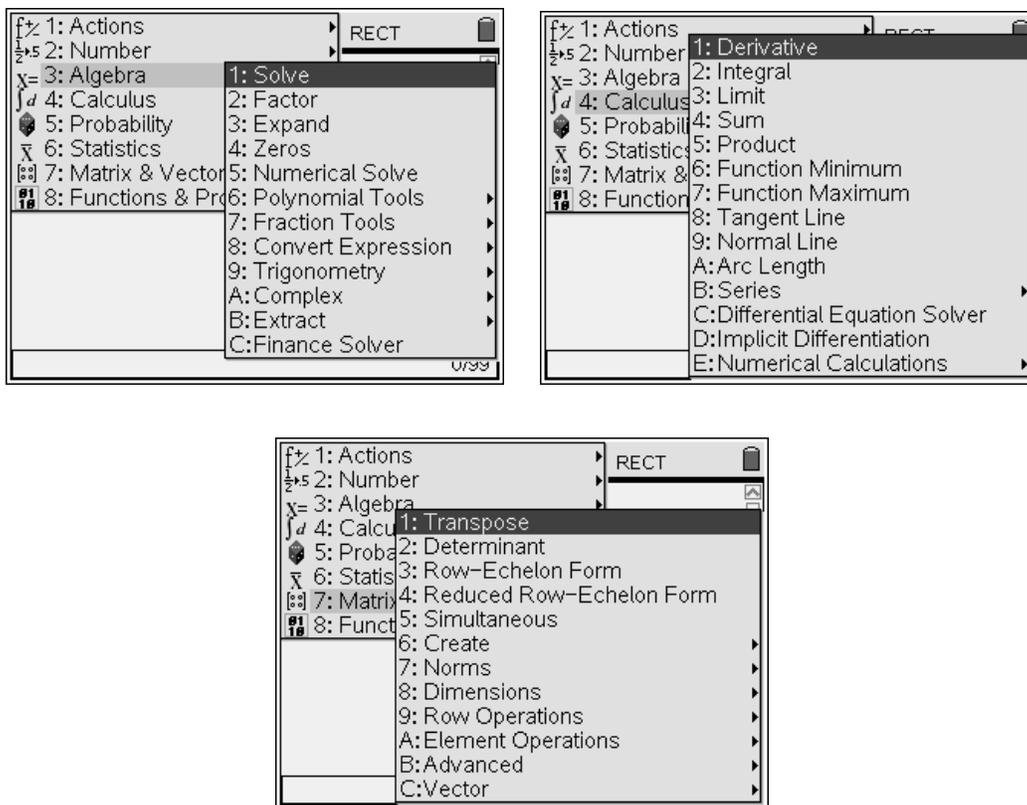


Figure 9

With these new features, we are confident that Texas Instruments really wants to update the CAS part of the calculator (but, unfortunately, Voyage 200 OS is stuck at OS 3.10 since July 2005...). Let us show examples of some of these new functions.

a) One can show that $\sin(5\theta) = 16\sin^5\theta - 20\sin^3\theta + 5\sin\theta$. Voyage 200 “texpand” command can be used but we will still have expressions involving the cosine function. Nspire CAS new “convert to sine” function (see figure 10) will show this easily.

b) Our students know that, in order to find the first 3 terms of the Taylor series of $\sin(x^8)$, it will be easier (and much more faster!) to simply substitute w par x^8 in the series of $\sin w$. With the Voyage 200 “Taylor” command, it takes a long time to compute this. Imagine what happens if someone tries to compute the Taylor series of $\sin(\cos(e^{\tan x}))$ with Voyage 200! This is caused by the fact that the Taylor command has to calculate all the derivatives of the function up to the order specified. But, with Nspire CAS, a few seconds and you get the answer (even for a big or-

der!). Laurent series were not available with Voyage 200 but Nspire CAS has a new “Generalized Series” function that can find this: see again figure 10. Note that the terms of a series are now written from the lowest power to the biggest one, unlike with Voyage 200.

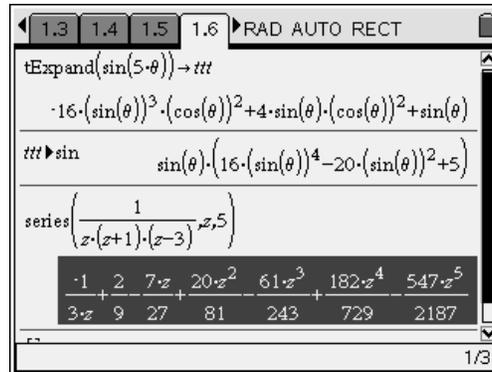


Figure 10

Example 2.6: using Nspire, our stat teachers would be happy to see that the Flash Application Stat/Lists Editor is now part of the basic functions of the calculator. All our students having a Voyage 200, we can do away with all basic tables commonly used in the classic approach when teaching statistics. Having a much faster processor can aid in various situations in statistics. Let's consider the classic problem of calculating a $(1-\alpha)100\%$ confidence interval for the real mean of a population based on a sample size of n elements and consider that the standard deviation in this population is unknown. A sample of 25 elements gives a sample mean of 34 and a sample standard deviation of 3.5. Assume that we have a Normal distribution for the statistic being studied and that we have a finite size population of 1200 elements. The general formula for calculating

the interval will be: $\mu \in \bar{X} \pm t_{\alpha/2; n-1} \frac{S}{\sqrt{n}} \sqrt{1 - \frac{n}{N}}$. With our data, this gives

$$\mu \in 34 \pm t_{0.025; 24} \frac{3.5}{\sqrt{25}} \sqrt{1 - \frac{25}{1200}} \Rightarrow \mu \in 34 \pm 1.4296$$

The real mean of the population is thus 34 plus or minus a margin of error of 1.4296 at a 95% level of confidence. Nspire, like the Voyage 200, has routines for calculating these intervals but will not take into account the finite population factor as we did in this example. One question of

interest here is to ask how many more elements should we have taking for the margin of error to be equal to 1.2 instead of 1.4296. In a classic stat course, this would be done using a Normal distribution approximation, but having these CAS calculators on hand, we can use the same formula using the Student distribution, which makes the problem much harder to solve since the Student's critical point depends also on the sample size.

Let's now solve this equation (see figure 11): $t_{0.975;n-1} \frac{3.5}{\sqrt{n}} \sqrt{1 - \frac{n}{1200}} = 1.2$

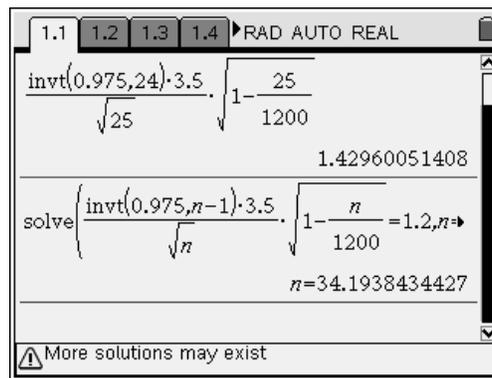


Figure 11

We can see that a sample size of 35 would give us the desired margin of error. Nspire handheld took 2 seconds to calculate this, the Voyage 200 would need almost 40 seconds; once again the processor speed makes a big difference. With Voyage 200 and Nspire calculators, students can do these calculations the same way Statistical software would do it.

Example 2.7: 3D plots are important for us, as for anyone teaching an advanced Calculus course. This next example is a problem from the textbook used in this course (Calculus, Concepts and Contexts from J. Stewart). Find the space curve representing the intersection of the cone $z = \sqrt{x^2 + y^2}$ with the plane $z = 1 + y$. This is not an easy problem for students and can only be done with well behaved equations. Here, letting $x = t$ and solving the 2 equations with this substitution gives the following parametric representation for the space curve:

$\vec{r}(t) = \left\langle t, \frac{t^2}{2} - \frac{1}{2}, \frac{t^2}{2} + \frac{1}{2} \right\rangle$. Students should have the reflex of checking these results using

appropriate graphing or CAS software (DERIVE, Maple, DPGraph...). Using Maple software

with the commands showed below, students can view the two surfaces, in figure 12a and plot the space curve they found, figure 12b. They can also easily rotate these 3D objects to get a good sense of what they're dealing with. Finding the right domain for each graph is also a good exercise for them.

```
> with(plots):
> plot3d({sqrt(x^2+y^2),1+y},x=-5..5,y=5..5,axes=frame, view=-2..15);
> spacecurve([t,t^2/2-1/2,t^2/2+1/2,t=-4..4],axes=boxed,color=red,thickness=3);
> a:=plot3d({sqrt(x^2+y^2),1+y},x=-5..5,y=-5..5,axes=frame,view=-2..15):
  b:=spacecurve([t,t^2/2-1/2,t^2/2+1/2,t=-4..4],axes=boxed,color=red,thickness=3):
  display(a,b);
```

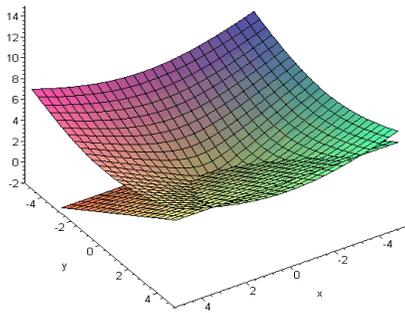


Figure 12a

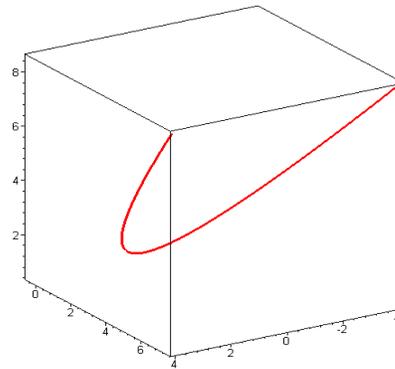


Figure 12b

With Maple, they can also merge the two graphs into one, allowing them to see if the space curve they found really fits the intersection of these two surfaces, see figure 12c. Adding 3D plotting to Nspire is a must to bring this product more in line with university level needs, given the fact that we already have other softwares doing the job.

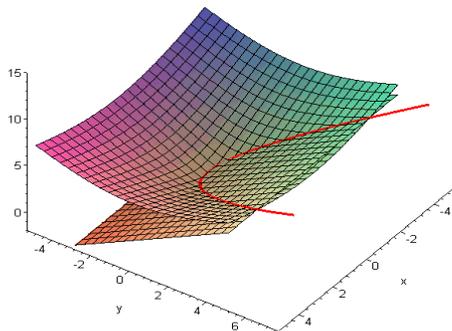


Figure 12c

3. Conclusion

When we started working on this conference project, one of our goal was to evaluate, in regards to our needs, this new Nspire CAS system (software and handheld), looking at the plus and minus and comparing with the Voyage 200 calculators used by all of our students. A big advantage of the new product is processor speed, which we illustrated in several examples in this paper. We wish we could have this processor in our Voyage 200. On the other hand, the lost of part of the differential equations facilities with the absence of 2D implicit plotting and 3D plotting in general is a big problem when we try to evaluate what benefit we would have in going from Voyage 200 to Nspire calculators. A lot of functionalities have been added to this new product but these are not things we would use. Going from the Voyage 200 keyboard to the one on Nspire handheld is a big drawback for us. We appreciate the menu shortcuts, the complete QWERTY keyboard, the function keys and the general speed of use of the Voyage 200. We also prefer the plotting environment in the Voyage 200. Lot's of features were added in Nspire in this respect but we feel this complicates working with plots. Furthermore, the handheld version is much harder to use when working with plots, and this is what our students would have on their desks. Nspire software is much easier to work with, but we already have other software available which teachers use. If we had to decide tomorrow on software for our math teaching, Maple would probably be the one chosen although engineering departments would prefer us working with MATLAB. So to answer the original question, no we would not use the new Nspire system. We wish to continue using the

Voyage 200 calculator and would like TI to consider further developing this product for a university level use. At our university, more than 3000 full-time students and over 100 of their professors are doing very well using this tool, many course curricula have been adapted for the general use of this CAS calculator. Since 1999, over 9000 students starting their studies in engineering have bought this CAS system (TI-92 Plus, TI-89 and now Voyage 200). Having a product, now, which meets our needs and, being so much used to working with it makes it very hard to look at a new product and, in a sense, start over while losing part of the ease of use we have with the Voyage 200 calculators.