

# Symbolic Hand-held Calculators in Teaching Engineering Mathematics

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## ABSTRACT

Since the 1999 fall semester, the TI-92 Plus or the TI-89 (scientific calculators with symbolic computation capabilities) has been compulsory for each new student entering our engineering school. We will illustrate the power of these calculators as well as the impact they have in our classrooms. Furthermore, we will present the work done to better support the efficient use of these calculators.

## 1. Introduction

In the 1996-1999 period, the TI-92 Plus and the TI-89 made their way into the classrooms and brought about many problems. In that period, these symbolic hand-held calculators from Texas Instruments were the only ones available with general algebraic computation capabilities. Today, other models can be found, for example the Hp 49g from Hewlett-Packard or the FX2.0 from Casio. Let's first look at what is special about these symbolic calculators. Up to 1996, a computer was necessary to perform general algebraic computations. By symbolic or algebraic computation, we mean the ability to manipulate symbols or variables and to apply algebraic rules. Consider the following classical example of algebraic simplification.

$$\frac{t^2 - 4}{t + 2} = \frac{(t-2)(\cancel{t+2})}{(\cancel{t+2})} = t - 2$$

The TI-92 Plus, which is used to produce the screen shots throughout this paper, can easily perform the same simplification as we can see from the following screen.

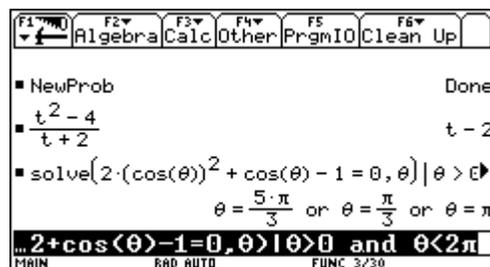


screen 1

Notice the entry or command line, with white characters over black. Above that, on the left, you will find the last expression entered and on the right, the answer or simplified expression. On the next screen, you will see that the following trigonometric equation was solved by the calculator using algebra instead of numerical methods.

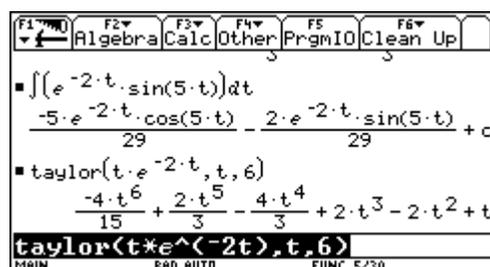
$$2 \cos^2(\theta) + \cos(\theta) - 1 = 0$$

Observe that the solutions of this equation were restricted to angles  $\theta$ , in radians, between 0 and  $2\pi$ .



screen 2

Using symbolic computations, these calculators can perform derivatives, integrals, Taylor series, etc. The following screen illustrates such calculations.



screen 3

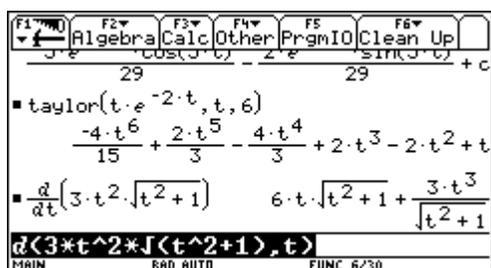
Of course, as more and more students brought this type of calculator in the classroom, it challenged how we taught mathematics. For example, some students couldn't understand why we would take 15 minutes to do a problem by hand, using the classical approach, when the calculator gave the same answer in 15 seconds! It was also becoming more difficult to design tests that would correctly assess

student learning. To some extent, and in an effort to minimize inequities, this led us to essentially supply answers to questions. For example:

Show that the derivative of  $f(t) = 3t^2\sqrt{t^2+1}$  is

$$f'(t) = \frac{3t(3t^2+2)}{\sqrt{t^2+1}}$$

It is interesting to notice that the calculator doesn't give the answer shown above, see screen 4. The student still has to perform some algebraic simplifications to get the required answer.



screen 4

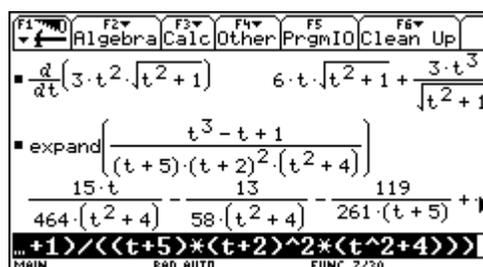
Discussions among faculty members are not always easy. Mathematics and science teachers can be conservative when it comes to change. Fortunately, at our university, a majority of math instructors had already begun using Computer Algebra Systems (CAS) and started having students work on more sophisticated or complex problems using DERIVE or MAPLE software. So, as a group and with the approval of our university, we decided to make it a requirement, as of the fall of 1999, that all new students entering an undergraduate engineering degree program obtain a TI-92 Plus or TI-89. They learn how to use them efficiently in their first math course (Calculus).

## 2. Impact on our teaching

The introduction of this hand-held technology forced us to re-assess and question our goals in teaching mathematics. This provoked many stimulating discussions among faculty members. What do we want our students to learn? What is important? In particular, what should they still be able to do manually? And, how do we adapt our assessment tools? Consequently, we re-examined course syllabi and replaced some tedious manual calculations and techniques with the development of problem solving skills and the intelligent use of the calculators. In our basic Calculus course, we now see less integration techniques and generally allow students to use the calculator for algebraic simplifications. A good example of what could be better left to the calculators is the classic problem of breaking down a rational function, like the one below, in a sum of simpler partial fractions.

$$\frac{t^3 - t + 1}{(t+5)(t+2)^2(t^2+4)}$$

The technique is quite simple to implement and understand, but calculations can be quite long. On the next screen, the calculator has done, in a few seconds, such a breakdown. Compare this with the amount of time necessary to do it manually...



screen 5

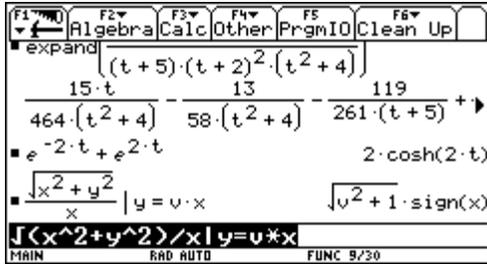
Traditionally, students would spend a lot of time practicing these calculations. By letting the calculator do part of the work, we now have more time in the classroom to show them how to use these machines efficiently. Students can spend more time exploring concepts instead of doing manual calculations. We must emphasize that using these machines well demands a good understanding of the underlying mathematics. For example, students will frequently be confronted with the task of showing that the answer they get, with the help of their symbolic calculators, is equivalent to the one found in their textbooks.

Using the TI can even be an excuse to introduce new subject matter. For example, we ask students to enter on their calculators, the next expression, a sum of conjugate exponentials:

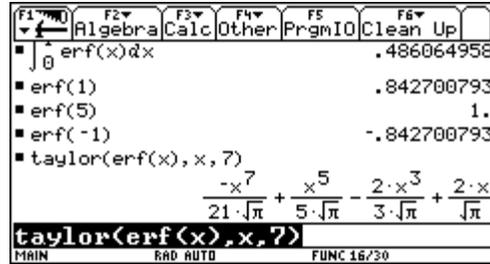
$$e^{-2t} + e^{2t}$$

The resulting simplification done by the TI, see screen 6, gives us the opportunity to introduce hyperbolic functions. When teaching first order differential equations, one technique demands substituting  $v \cdot x$  for  $y$ , resulting in an expression depending only on  $v$ . Students usually have no problem obtaining  $\sqrt{1+v^2}$  manually from the expression  $\frac{\sqrt{x^2+y^2}}{x}$  with this substitution. Using the calculator

(screen 6) to verify this simplification, we have to explain the meaning of this new (for them) function,  $\text{sign}(x)$ . It is a good opportunity to let them know that the result they obtain manually was right only if we consider the variable  $x$  to be positive!



screen 6



screen 8

As an added bonus, since all the students have on their desks this powerful calculator, they become more active during lectures. The use of the calculator, as we have seen from the examples above, is a source, in itself, of new math.

We can now explore, with our students, topics or concepts and illustrate them with more ease. Let's consider a classical function for many engineers, the error function:

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt$$

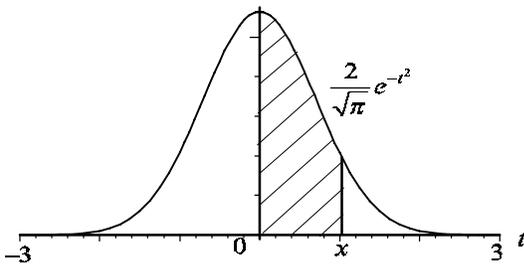
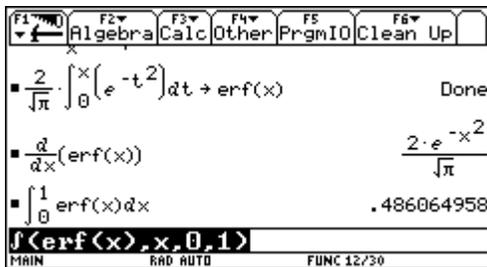


fig. 1. representing the error function

With the calculator, we can define this function and store it into memory. Having done this, we can ask our TI to do various computations with this function, treating it like an ordinary function. On screen 7, below, we differentiate and integrate erf(x).



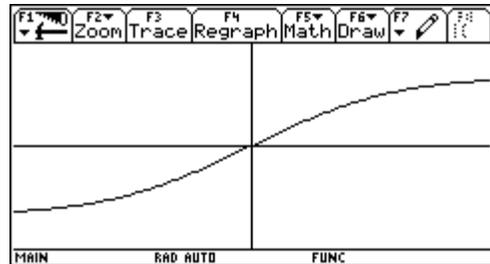
screen 7

We can evaluate erf(x) for different values of x; the calculator uses numerical integration for each evaluation. We can even have the calculator find the Taylor series for this function (screen 8).

This is a good opportunity to explore with our students their understanding of some classical Calculus results. For example, we ask them if the following result is always true, and to explain why:  $\text{erf}(-x) = -\text{erf}(x)$ . Or, we can ask them why one shouldn't be surprised that the derivative of erf(x) is as seen on screen 7:

$$\frac{d}{dx} \text{erf}(x) = \frac{2}{\sqrt{\pi}} e^{-x^2}$$

Of course, these calculators have all the usual features of modern graphic calculators and more, so we can ask it to graph our function erf(x). This will take a few minutes since each point in the graph will be found through numerical integration. You can see below, on screen 9, the resulting graph, each axis going from -1,5 to 1,5.

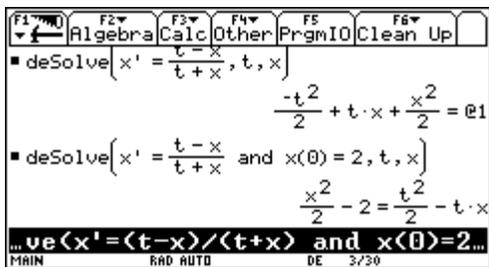


screen 9

Looking at this graph, we expect our students to be able to answer the following question without using their calculators:

$$\int_{-2}^2 \text{erf}(x) dx = ?$$

Another familiar math subject for engineering students is Differential Equations. Once again, the TI can be very useful. The calculator can solve, algebraically, some basic first and second order equations. Screen 10 gives an example of the calculator solving a first order differential equation: a general solution and a particular one given the initial condition.

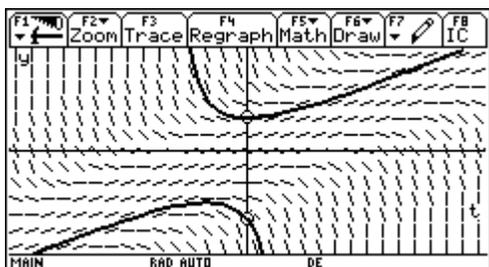


screen 10

Algebraic solutions to general first order differential equations can be a tricky business. The TI isn't able yet\* to solve, for example, the equation below.

$$\frac{dx}{dt} = 1 - (t-x)^2$$

Students learn that this equation is easily solved, manually, by using the substitution  $v = t - x$ , which transforms it into a separable form. Although the calculator can't find an exact solution, it can solve it numerically. On the following screen, you will find a direction field for the above equation. Two integral curves are also shown for initial conditions  $x(0)=1$  and  $x(0)=-2$ ; they were determined using a Runge-Kutta method.

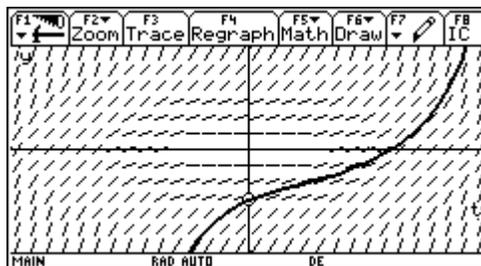


screen 11

We can now easily investigate differential equations that have no solution in terms of elementary functions. A classical example is given by the following equation.

$$\frac{dx}{dt} = t^2 + x^2 \text{ with } x(0) = -1$$

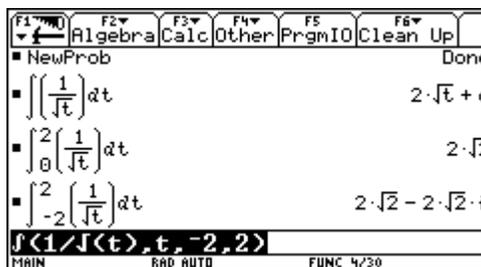
On screen 12, below, you will find the direction field for this equation, illustrating the behaviour of the integral curves, as well as a solution curve for the initial condition mentioned above.



screen 12

Assessment tools must be adapted to the use of these new tools. For example, part of the test on derivatives is done without a calculator. We wish to ensure that students master basic rules and are able to do simple problems manually. On the other hand, in optimization problems, we encourage them to use the calculators as much as possible, thus concentrating their efforts on the modelling aspects of the problem and on the correct interpretation of the results.

We have to be more specific when writing exam questions. We don't see a problem in asking students to solve a problem using a specified technique and having them give a detailed outline of their solution. The fact that the calculator can give them a direct answer can even be a disadvantage. If the calculator uses a different technique in solving the specified problem, the answer found can be quite different, but equivalent, to the one found manually by the student. Furthermore, using a powerful tool can bring up some surprising results. On the following screen, the calculator evaluates a definite integral and gets a complex number as an answer. This is correct because the calculator is set to have the variable  $t$  take real or complex values.



screen 13

In order to verify that they have learned the proper use of the power of their symbolic calculator, they should be also tested on more challenging problems.

Finally, it should be noted that we still have students use CAS on computers when printouts are needed or if the problems are better explored with more powerful tools.

### 3. The web site

Implementing the mandatory use of the TI-89/92 Plus calculators in the fall of 1999 wasn't an easy task. We had

\* Texas Instruments regularly updates the Mathematics Software running on these machines. With these calculators, having flash ROM technology, users can download new versions on their calculator. It is therefore possible that in a future version, the TI will be able to solve this equation.

to make sure that the appropriate equipment was available and that all instructors were capable of using this equipment.

We realized that there were few resources available in French, on the web, to support the use of these calculators. As a result of our experience, in the summer 2000 we collected questions that students or instructors often asked us on the use of these calculators. We then designed (with the assistance of two colleagues, Michel Beaudin and Chantal Trottier) the web site <http://www.seg.etsmtl.ca/ti> ,

which provides quick and easy access to information on the use of these calculators. Its guided tour, FAQ, and different documents (such as how to solve an equation, how to draw a 3d graph, how to use matrices...) were designed to meet the specific needs of our students and faculty members.

We have translated part of our web site into English, see <http://www.seg.etsmtl.ca/ti/English/> , to give you an idea of the work that was accomplished; figure 2 is a screen shot of the welcome page.

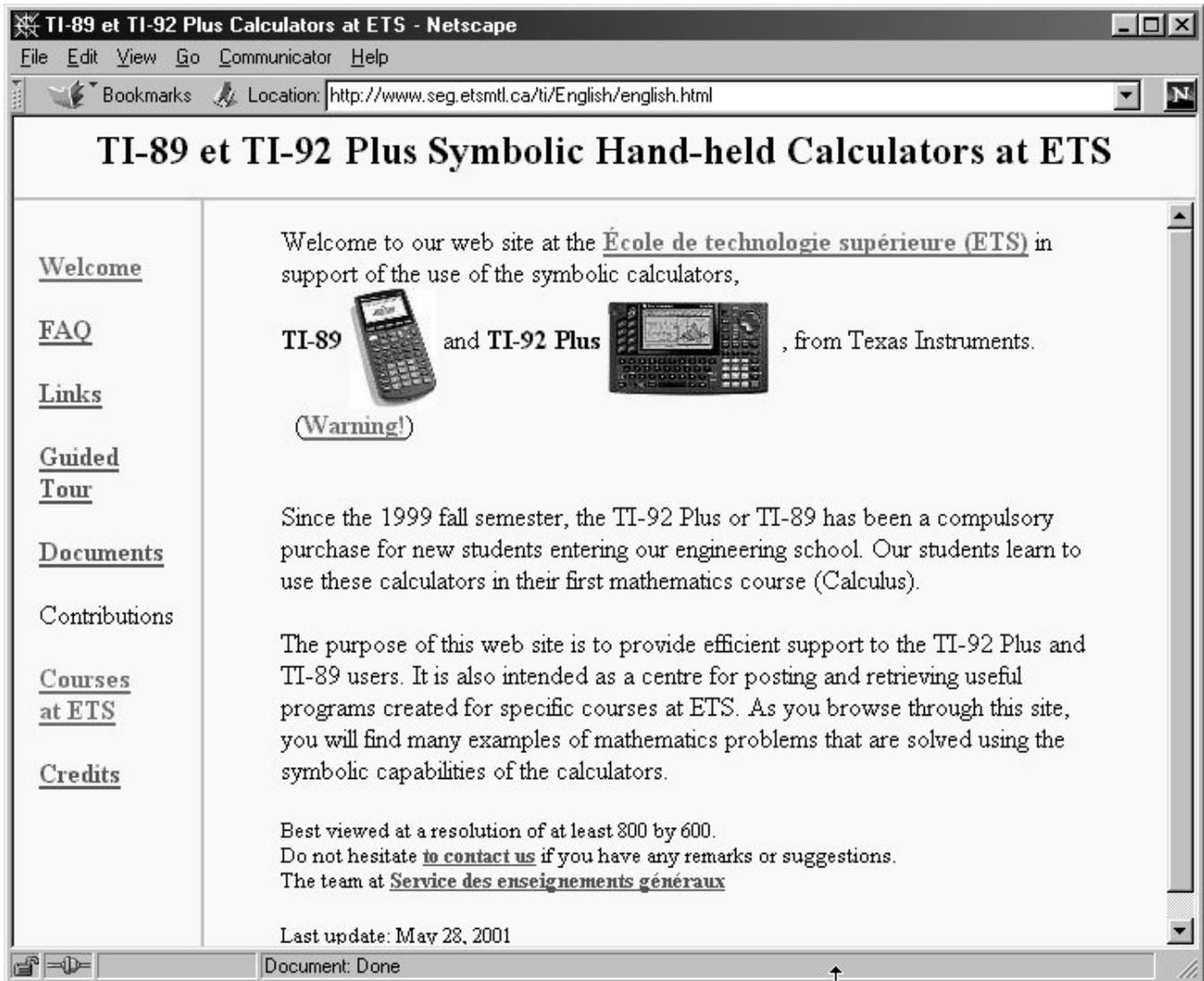


fig. 2. Welcome page of our web site.

The questions that we collected were classified into two groups. If a short answer was all that was necessary, it was put in the **FAQ** section. For instance, *How do I stop a calculation?* is listed in the FAQ. If the question demanded a more detailed explanation, such as *How do I solve a system of equations?* or *How do I enter and use matrices?* it was put in the **Documents** section of our site.

Another aspect of our experience was extending the use of the calculators to more courses at our university. We met with interested professors from the four engineering departments to present the calculator's potential, gave workshops, and informed them that their students had this technology at hand. To help stimulate discussion about this technology within the departments, and the changes it would

inevitably bring to teaching, we hired four students, one per department, to work on small TI application modules. Each of these modules was designed to meet the needs of one or two courses in each of the four departments. For example, under the supervision of two professors from the Civil Engineering department, a student produced the links associated to **CTN-208** and **CTN-408** (courses on Analysis of Structures and Mechanics of Materials) in the **Courses at ETS** page of our site. His job was to write a set of TI programs and/or commands that can be downloaded from our site onto the TI-89/92 Plus. He also had to produce the corresponding user's guide. Now, CTN-208 and CTN-408 students download these files to their TI and use them in these courses.

#### 4. Conclusion

What began in our mathematics classes is now spreading to other courses. More and more of our department's science instructors now have their own TI calculator and have started to integrate its use in their teaching. The same is happening in the engineering departments. So far the results are positive. Students and mathematics instructors have adapted to this new technology. The fact that our experience stimulated discussions and focused attention on the fundamentals of teaching makes this project a success. The hands-on approach made available by this technology makes for a more dynamic classroom as the students participate more actively in the exploration of mathematical ideas. We have noticed that the use of these symbolic calculators generates more exchanges between students. We see using the available technology as one way to help

students learn. There's still resistance from some faculty members and on-going discussions on how to adapt our teaching to take into account this technology. We must continue to put forward the benefits of integrating computer algebra in our teaching and consider it important to have these discussions.

In view of our experience in integrating this technology in our classrooms, we can offer some general tips:

- take the time necessary to ensure that instructors are at ease with the technology, offer incentive and support;
- don't try to keep all the classical topics and manual calculations; you have to make room for new technology;
- encourage students to use the technology intelligently, do calculations with them in the classroom, use the ViewScreen to show them what you are doing;
- show students common mistakes that are made with the calculator, illustrate its limitations;
- insist on the comprehension and the importance of mathematical syntax;
- have them explore, try and explore, and try...

One must realize that this technology hasn't changed our math courses that much. More emphasis is now put on concepts instead of manual skills. Plus, we can explore more complex problems, which are probably closer to what students will encounter in their engineering classes. As a final note, we are now teaching to what may be called "the Nintendo generation". They appreciate working with these machines and realize that they need to really understand the mathematics behind them to make efficient use of the technology. They ask more questions, do more exploration than they used to and are happy doing it.