

TIME 2014
Technology in Mathematics Education
July 1st - 5th, Krems, Austria

*Technology in Teaching Mathematics:
Looking Back and Looking Forward*

Keynote Lecture

The students we now have in 2014 were almost born with technology in their hands. Numerical and graphical calculators are accepted tools on every student's desk. But they also have smartphones, tablets, ipads, with even more powerful apps, always at their sides. How does this change math education? What can we still expect them to be able to do "by hand" and where will they need technology to support them? Have we gained something while encouraging them to use all these tools?

We will show examples of math calculations that used to be done manually 30 or 40 years ago and that are accepted today as being done via technology. Today's students will smile if we attempt to explain that these calculations used to require a lot of time and paper. Looking at the way technology has evolved in the last 15 years, we believe we should be asking ourselves which math operations will make students smile in 30 years from now, thinking of how we did these calculations in 2014. Since numerical calculations are now mainly done with technology, we need to consider which level of algebra students really need to acquire, since calculators, apps and software can do more and more of these operations for them. Computer Algebra Systems (CAS) are easily available. Shouldn't we be showing them how to better use this technology? To be aware of its limits and caveats. Examples will be presented to illustrate this.

At our university, all engineering students are required to have a CAS calculator. This has been mandatory now for 15 years. We will discuss how this has changed the way we teach some of our topics and show examples of how technology has allowed us to continue to cover the same general curriculum, discarding some manual calculations and having students explore and work even more mathematics or more challenging problems. We will also present how students themselves evaluate this technology in their learning environment and how good they consider their teachers are in teaching and using this CAS environment in the classrooms.

Technology in Teaching Mathematics: Looking Back and Looking Forward

Overview

- ▣ Context! Our experience with CAS at ETS
- ▣ What do we teach? How?
- ▣ Looking back...
- ▣ How technology has changed our teaching
- ▣ Looking forward..., What should/could change...
- ▣ Conclusion

About ETS : École de technologie supérieure



- Engineering school in Montréal, Québec, Canada
- We hosted [TIME-2004](#) and [ACA 2009](#) (Applications of Computer Algebra) at our university
- Our students are mainly graduates from college technical programs
- More info: http://www-eng.etsmtl.ca/ets_in_numbers.html

About ETS : École de technologie supérieure



- Almost 1 out of 4 engineers in Québec comes from ÉTS
- More than 7600 students (5600 at undergraduate level)
- 1500 new students each year
- All maths teachers and students have the same calculator and textbooks

About ETS : ours tools

- 1999: TI-92 Plus CAS handheld
 - CAS calculators are mandatory since 1999
 - 2002 : TI Voyage 200
 - Other software (Derive, Maple, Matlab, DPGraph, Geogebra) are used by some teachers.
 - 2011 : TI-Nspire CAS CX
- Only CAS calculators are allowed during math and science exams (no laptop).



What do we teach? How?

- ▣ CAS handheld calculators are very well integrated in our curriculum and courses (see previous conferences of [Gilles Picard](#) and/or [Michel Beaudin](#))
- ▣ Next September, we will have at least 108 different groups or classes, in math and science (29 of these groups will be first semester course in mathematics)
- ▣ over 40 different teachers, lecturers and assistants, will be in charge of these groups
- ▣ each course, lasting 13 weeks including midterms (but not counting final exams) have 3 hours a week for theory with a lecturer or teacher and 3 hours/week for tutorials/lab/practical work with the same lecturer or with a teaching assistant.
- ▣ Math topics include: pre-calculus, calculus of one and several variables, basic linear algebra, differential equations, probability and statistics, discrete math, Fourier analysis etc.

Looking back...

- ▣ before 1997, we would roll in classes chariots with projector and computer, to show DERIVE or Maple software
- ▣ celebrating this summer 22 years of ACDCA conferences and 20 years ago the first DERIVE conference in Plymouth UK
- ▣ my first conference in 2001 was in Albuquerque New Mexico, ACA'2001 Applications of Computer Algebra (Education meets Computer Algebra session) with my colleagues Michel Beaudin and Kathleen Pineau
- ▣ at ÉTS, the last 15 years of mandatory CAS calculator for all students have been quite an experience
- ▣ Let's go even further back...

Looking back...

What are we calculating here « by hand »?

The image shows two handwritten long division calculations. The left calculation is for the square root of 176.68, and the right calculation is for the square root of 13.292.

Left Calculation (Square Root of 176.68):

$$\begin{array}{r} 176.68 \\ \sqrt{} \\ \underline{076} \\ 69 \\ \underline{768} \\ 524 \\ \underline{24400} \\ 23841 \\ \underline{55900} \\ 53164 \\ \underline{2736} \end{array}$$

Right Calculation (Square Root of 13.292):

$$\begin{array}{r} 13.292 \\ \sqrt{} \\ \underline{4} \\ 23 \\ \underline{262} \\ 2649 \\ \underline{26582} \end{array}$$

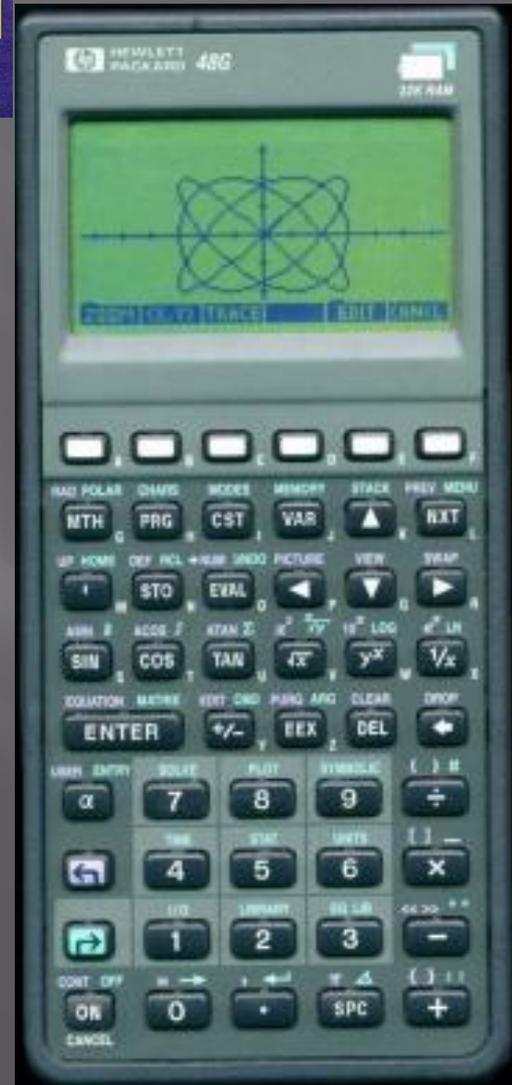
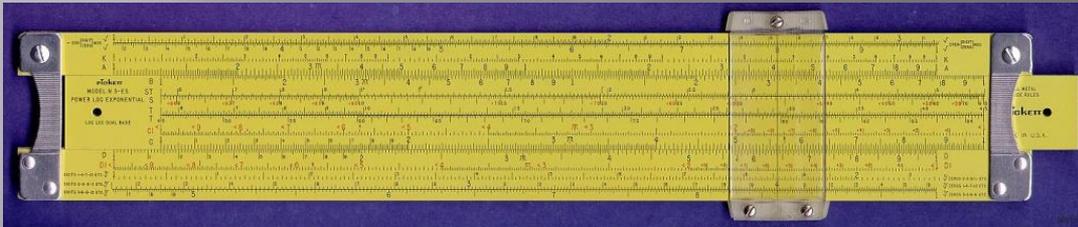
Of course, we are calculating the square root of 176,68

Looking back...

calculating trig values without a calculator

θ Deg.	θ Rad.	Sin θ	Cos θ	Tan θ	Cot θ	Sec θ	Csc θ		
27°00'	0.4712	0.4540	0.8910	0.5095	1.9626	1.122	2.203	1.0996	63°00'
10'	0.4741	0.4566	0.8897	0.5132	1.9486	1.124	2.190	1.0966	50'
20'	0.4771	0.4592	0.8884	0.5169	1.9347	1.126	2.178	1.0937	40'
30'	0.4800	0.4617	0.8870	0.5206	1.9210	1.127	2.166	1.0908	30'
40'	0.4829	0.4643	0.8857	0.5243	1.9074	1.129	2.154	1.0879	20'
50'	0.4858	0.4669	0.8843	0.5280	1.8940	1.131	2.142	1.0850	10'
28°00'	0.4887	0.4695	0.8829	0.5317	1.8807	1.133	2.130	1.0821	62°00'
10'	0.4916	0.4720	0.8816	0.5354	1.8676	1.134	2.118	1.0792	50'
20'	0.4945	0.4746	0.8802	0.5392	1.8546	1.136	2.107	1.0763	40'
30'	0.4974	0.4772	0.8787	0.5430	1.8418	1.138	2.096	1.0734	30'
40'	0.5003	0.4797	0.8774	0.5467	1.8291	1.140	2.085	1.0705	20'
50'	0.5032	0.4823	0.8760	0.5505	1.8165	1.142	2.074	1.0676	10'
29°00'	0.5061	0.4848	0.8746	0.5543	1.8040	1.143	2.063	1.0647	61°00'
10'	0.5091	0.4874	0.8732	0.5581	1.7917	1.145	2.052	1.0617	50'
20'	0.5120	0.4899	0.8718	0.5619	1.7796	1.147	2.041	1.0588	40'
30'	0.5149	0.4924	0.8704	0.5658	1.7675	1.149	2.031	1.0559	30'
40'	0.5178	0.4950	0.8689	0.5696	1.7556	1.151	2.020	1.0530	20'
50'	0.5207	0.4975	0.8675	0.5735	1.7437	1.153	2.010	1.0501	10'
30°00'	0.5236	0.5000	0.8660	0.5774	1.7321	1.155	2.000	1.0472	60°00'
10'	0.5265	0.5025	0.8646	0.5812	1.7205	1.157	1.990	1.0443	50'
20'	0.5294	0.5050	0.8631	0.5851	1.7090	1.159	1.980	1.0414	40'
30'	0.5323	0.5075	0.8616	0.5890	1.6977	1.161	1.970	1.0385	30'
40'	0.5352	0.5100	0.8601	0.5930	1.6864	1.163	1.961	1.0356	20'
50'	0.5381	0.5125	0.8587	0.5969	1.6753	1.165	1.951	1.0327	10'
31°00'	0.5411	0.5150	0.8572	0.6009	1.6643	1.167	1.942	1.0297	59°00'
10'	0.5440	0.5175	0.8557	0.6048	1.6534	1.169	1.932	1.0268	50'
20'	0.5469	0.5200	0.8542	0.6088	1.6426	1.171	1.923	1.0239	40'
30'	0.5498	0.5225	0.8526	0.6128	1.6319	1.173	1.914	1.0210	30'
40'	0.5527	0.5250	0.8511	0.6168	1.6212	1.175	1.905	1.0181	20'
50'	0.5556	0.5275	0.8496	0.6208	1.6107	1.177	1.896	1.0152	10'
32°00'	0.5585	0.5299	0.8480	0.6249	1.6003	1.179	1.887	1.0123	58°00'
10'	0.5614	0.5324	0.8465	0.6289	1.5900	1.181	1.878	1.0094	50'
20'	0.5643	0.5348	0.8450	0.6330	1.5798	1.184	1.870	1.0065	40'
30'	0.5672	0.5373	0.8434	0.6371	1.5697	1.186	1.861	1.0036	30'
40'	0.5701	0.5398	0.8418	0.6412	1.5597	1.188	1.853	1.0007	20'
50'	0.5730	0.5422	0.8403	0.6453	1.5497	1.190	1.844	0.9977	10'
33°00'	0.5760	0.5446	0.8387	0.6494	1.5399	1.192	1.836	0.9948	56°00'
10'	0.5789	0.5471	0.8371	0.6536	1.5301	1.195	1.828	0.9919	50'
20'	0.5818	0.5495	0.8355	0.6577	1.5204	1.197	1.820	0.9890	40'
30'	0.5847	0.5519	0.8339	0.6619	1.5108	1.199	1.812	0.9861	30'
40'	0.5876	0.5544	0.8323	0.6661	1.5013	1.202	1.804	0.9832	20'
50'	0.5905	0.5568	0.8307	0.6703	1.4919	1.204	1.796	0.9803	10'
34°00'	0.5934	0.5592	0.8290	0.6745	1.4826	1.206	1.788	0.9774	56°00'
10'	0.5963	0.5616	0.8274	0.6787	1.4733	1.209	1.781	0.9745	50'
20'	0.5992	0.5640	0.8258	0.6830	1.4641	1.211	1.773	0.9716	40'
30'	0.6021	0.5664	0.8241	0.6873	1.4550	1.213	1.766	0.9687	30'
40'	0.6050	0.5688	0.8225	0.6916	1.4460	1.216	1.758	0.9657	20'
50'	0.6080	0.5712	0.8208	0.6959	1.4370	1.218	1.751	0.9628	10'
35°00'	0.6109	0.5736	0.8192	0.7002	1.4281	1.221	1.743	0.9599	55°00'
10'	0.6138	0.5760	0.8175	0.7046	1.4193	1.223	1.736	0.9570	50'
20'	0.6167	0.5783	0.8158	0.7089	1.4106	1.226	1.729	0.9541	40'
30'	0.6196	0.5807	0.8141	0.7133	1.4019	1.228	1.722	0.9512	30'
40'	0.6225	0.5831	0.8124	0.7177	1.3934	1.231	1.715	0.9483	20'
50'	0.6254	0.5854	0.8107	0.7221	1.3848	1.233	1.708	0.9454	10'
36°00'	0.6283	0.5878	0.8090	0.7265	1.3764	1.236	1.701	0.9425	54°00'
		Cos θ	Sin θ	Cot θ	Tan θ	Csc θ	Sec θ	θ Rad.	θ Deg.

Looking back...

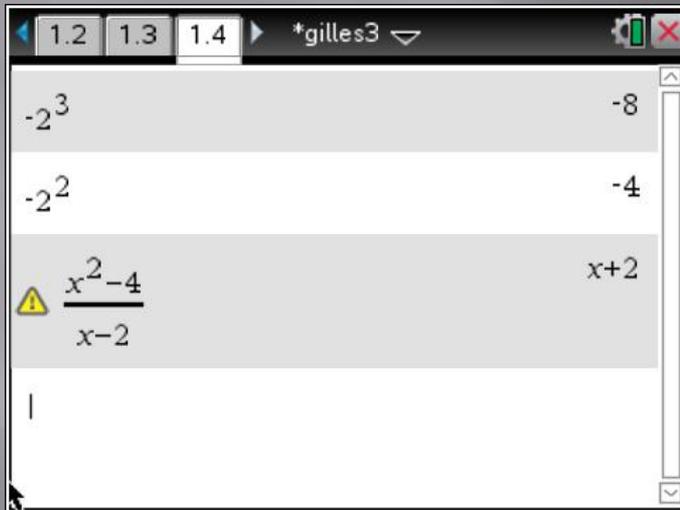


Today

- ▣ we accept working with technology for numerical calculations (or not...)
- ▣ basic arithmetic: 8×7 or $28/4$ which students should know...
- ▣ this is necessary for what purpose? Apart for the obvious, we need this for algebra calculations, for example to factorise $x^2 + 15x + 56$ or derive $8x^7$
- ▣ we do want them to calculate simple problems by hand and this should always be the case
- ▣ in pre-calculus and in the first course on calculus, part of the exams at ETS are without calculators

Today

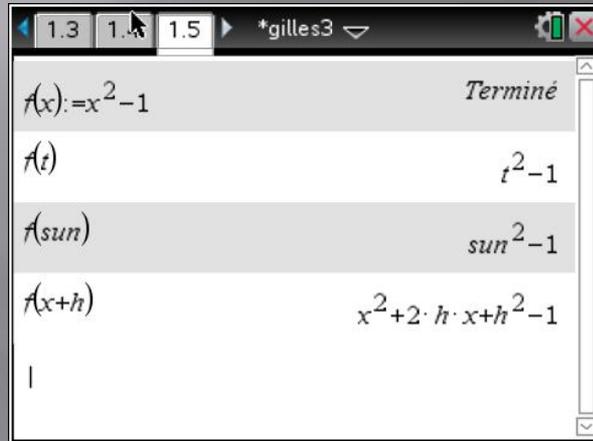
- many students arriving at ETS are lacking basic math knowledge and have poor numerical skills
- basic errors and why technology can help (or not)



Technology cannot help here if they don't know the priority of operations. But a CAS calculator can motivate them in the second example to understand where the simplification came from

Today

- Motivating basic algebra with the aid of CAS



A screenshot of a CAS interface showing a list of function evaluations. The window title is '*gilles3'. The interface displays the following table:

$f(x) := x^2 - 1$	<i>Terminé</i>
$f(t)$	$t^2 - 1$
$f(\text{sun})$	$\text{sun}^2 - 1$
$f(x+h)$	$x^2 + 2 \cdot h \cdot x + h^2 - 1$

- Teaching the concept of function
- You can ask them, what happened with the last result, can they explain it...

Today

- ▣ Solving non-linear system of equation
- ▣ A physic colleague came to see me, complaining that the CAS calculator (the Voyage 200 at that time), was not able to solve this system of equation:

$$(1) \sqrt{2 \left[\frac{k}{2} - kA \ln 1 + c \right]} = 0$$

$$(2) \sqrt{2 \left[\frac{k f^2}{2} - kA \ln f + c \right]} = 2 \sqrt{2 \left[\frac{k 2^2}{2} - kA \ln 2 + c \right]}$$

$$(3) \sqrt{2 \left[\frac{k 16^2}{2} - kA \ln 16 + c \right]} = 29$$

Today

- True, Nspire-CAS is not able to solve the system yet, but if you square the equations, you get an answer
- Interesting here to note that squaring the 3 equations is exactly how this teacher started his 2 pages of manual solution for this problem
- Technology will not do everything for the user, we need to learn how to work with it

$$\text{solve} \left(\begin{cases} \sqrt{2 \cdot \left(\frac{k}{2} - k \cdot a \cdot \ln(1) + c \right)} = 0 \\ \sqrt{2 \cdot \left(\frac{k}{2} \cdot 8^2 - k \cdot a \cdot \ln(8) + c \right)} = 2 \cdot \sqrt{2 \cdot \left(\frac{k}{2} \cdot 2^2 - k \cdot a \cdot \ln(2) + c \right)} \\ \sqrt{2 \cdot \left(\frac{k}{2} \cdot 16^2 - k \cdot a \cdot \ln(16) + c \right)} = 29 \end{cases}, \{a, k, c\} \mid k > 0 \text{ and } c < 0 \text{ and } a < 0 \right)$$

$$\sqrt{-2 \cdot (4 \cdot a \cdot k \cdot \ln(2) - c - 128 \cdot k)} = 29 \text{ and } \sqrt{-2 \cdot (3 \cdot a \cdot k \cdot \ln(2) - c - 32 \cdot k)} = 2 \cdot \sqrt{-2 \cdot (a \cdot k \cdot \ln(2) - c - 2 \cdot k)} \text{ and } \sqrt{2 \cdot c + k} = 0$$

$$\text{solve} \left(\begin{cases} 2 \cdot \left(\frac{k}{2} - k \cdot a \cdot \ln(1) + c \right) = 0 \\ 2 \cdot \left(\frac{k}{2} \cdot 8^2 - k \cdot a \cdot \ln(8) + c \right) = 8 \cdot \left(\frac{k}{2} \cdot 2^2 - k \cdot a \cdot \ln(2) + c \right) \\ 2 \cdot \left(\frac{k}{2} \cdot 16^2 - k \cdot a \cdot \ln(16) + c \right) = 29^2 \end{cases}, \{a, k, c\} \mid k > 0 \right)$$

$$a = \frac{-51}{2 \cdot \ln(2)} \text{ and } c = \frac{-841}{918} \text{ and } k = \frac{841}{459}$$

Today

- ▣ Probability and statistics: our basic course doesn't use classic tables anymore (Binomial, Poisson, Normal, Student's, etc). All our students have a calculator with all this statistic functions integrated and much more (data analysis, confidence intervals, hypothesis tests, regression). Our teaching has to adapt to this evolution of technology...
- ▣ Let's do more statistics and more applied problems with correct interpretation of results, and less manual calculations

Today

- ▣ Let's talk about the need for applications: very early in math education we consider application of basic concepts.
- ▣ For example consider a situation where you want to buy online tickets for an upcoming show. Let's say it costs 4\$ (internet buying fee) plus 2\$ per ticket: So you create this function for cost $C(n) = 4 + 2n$
- ▣ What's the cost for 5 tickets $C(5) = 4 + 2 \times 5 = ?$
- ▣ This is very elementary math and numerical calculation, so everyone should be able to get the correct cost...

Today

- Very good article by John Wilson:
- <http://www.casinoenterprisemanagement.com/articles/april-2010/mathbox-20-view-soapbox>

John Wilson

The Slot Mathemagician
Peterborough, Ontario, Canada (Ontario, Canada) | Writing and Editing

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John Wilson's Overview

Current	Writer at BNP Media The Slot Mathemagician at ICS Gaming Acting Captain at Peterborough Fire Services
Past	Technology Editor at Casino Enterprise Management Freelance Writer at Game Time International Columnist at Slot Tech Magazine
Education	Sir Sandford Fleming College
Connections	168 connections

John Wilson's Experience

Writer
BNP Media
Privately Held; 201-500 employees; Publishing industry
June 2012 – Present (2 years 1 month)

The Slot Mathemagician
ICS Gaming
December 2003 – Present (10 years 7 months) | Peterborough, Ontario
I am the Slot Mathemagician, published in 9 magazines around the globe with over 100 articles published.

My articles center on slot math theory, game design, slot systems and CRM/BI. I attend gaming conferences where I am frequent moderator and provide a three-hour session on slot math.

Today

A good article on how technology and math education have evolved over the last 30-40 years

ARTICLES

CASINOFEST™ 10

Mathbox 20: The View from the Soapbox

Article Author	Publish Date	Article Tools
John Wilson	March 31, 2010	

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About the same time I started high school, affordable pocket calculators were coming onto the market. My first calculator was quite simple, an eight-digit Texas Instruments with a red LED display and a magnifying lens over each of the digits. It provided the four basic functions and nothing more. I suppose that it's not surprising I had one—I was a technology junky even before it was fashionable. From that first basic device, I soon moved on to more advanced calculators with trigonometric functions, graphing capabilities, and finally, programmable in BASIC.

Harbingers of doom? Perhaps. Not all was well in technology land back in those days. Teachers warned that these diminutive pocket calculators were, in fact, harbingers of doom. They would make us lazy, turn our brains into mush, and render us unable to perform the most rudimentary of operations without them.

I was, however, able to convince my math teacher to allow me to use this evil device to check my work and verify my answers. This, I believed, was logical and sound. If I had incorrect answers, I could go back and redo them until I had done them correctly. Otherwise, I was proceeding under the assumption that my work was correct. I wanted to use technology as a tool to verify my work and help me learn. This would surely be better than waiting

until class the following day to find out that I had the wrong answer, when I had neither the time nor inclination to redo the work. By that time we would have moved onto another lesson, and I would have an erroneous understanding of the material we were continuing to build upon.

Today

Now let's go back to my earlier problem of buying 5 tickets at 2\$ each. This does seem to be a problem for some primary math school teachers...

Last week I came across an article discussing mathematics in England. The article declared that primary school math teachers were failing to attain the standard of arithmetic expected of 11-year-olds, according to new research. Furthermore, on average, the teachers answered only 45 percent of the questions correctly.

One such question was included in the article: What is $4 + 2 \times 5$? Apparently it's a difficult question, and one that few answered correctly. I seriously doubt, however, that anyone reading this article would be unable to answer it correctly. Yet, the article claims that a television station asked teachers this question while conducting their own research for a feature documentary. Only 20 percent of teachers could answer the question. Even a television show reported that the answer to the question was 30 rather than 14, the correct answer.

What I found most interesting was the ensuing discussion on the webpage that reported the story. The conversation between readers and the comments they posted were startling. Among my favorites: "Actually if addition took priority it would be 40, not 30."

The only way I can get the answer to be 40 is to rewrite the question as $4 \times 5 \times 2$. If the question had been misread this way, I could understand it. And misreading the question actually makes more sense than most of the posts that followed. However, this blogger did know that addition was involved, so I am unsure how exactly he obtained his answer. Nonetheless, he was confident enough in his work to post a reply to the article.

The next response, I believe, is more typical of the current understanding of mathematics: "A question so ambiguous is a poor test of competence. One writer comments they used to use it as a trick question."

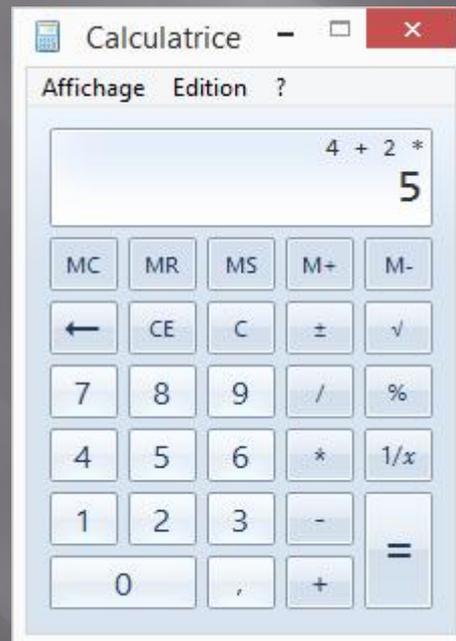
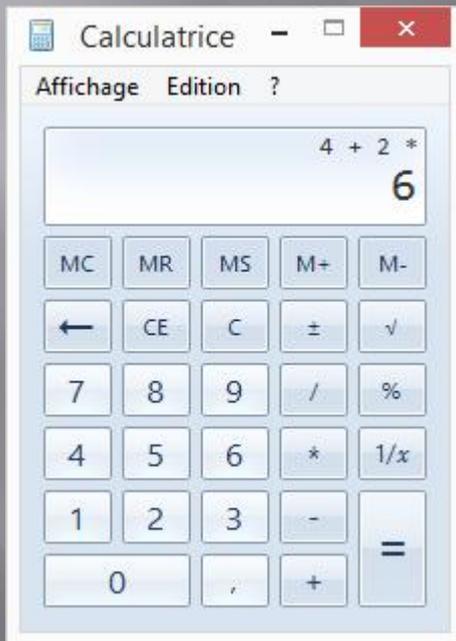
How could this possibly be construed as a trick question? This question is simple and basic in nature, testing for the simplest of mathematical rules: the order of operations. It is far from ambiguous. In fact, it would be difficult to have a simpler, more direct question.

This commenter at least recognized the point of the question:

"It is important to understand the order of mathematical operations when doing algebra, programming or scientific calculations. Without this understanding you would get into an unholy muddle quite quickly. Understanding the order of calculation is not really important for an 11-year-old as these types of calculations are usually beyond them. You may note that if the mathematical operations order were so important, then 60 percent of calculators should be considered faulty. For example, those using Microsoft Windows, as an OS, will note that there are two calculator options built in. One option is called 'standard' and the other 'scientific.' For the example given in the article, the standard calculator option would answer 30 and the scientific option would give 14. If an 11-year-old doesn't know this then it really is of no consequence as most adults don't know the answer either, teacher or otherwise."

Today

If you take the basic Window's calculator and do the calculation, you will find the wrong answer (30 after hitting the equal sign). Of course, this happens because this a very basic calculator, with no stack for intermediate results. In this case, after hitting the multiply, the calculator did the first operation showing 6 for result. Then hitting the 5, this makes the intermediate result disappear. Of course, this shows that technology can give wrong answers if you don't know how to ask the right questions... Of course the scientific version wil give the correct answer, 14.



Today

In 2010, this study done in UK was picked up by many media, for example this article in The Guardian:

<http://www.theguardian.com/education/2010/feb/14/primary-teachers-fail-maths-tests>

A test comprising "27 straightforward maths questions" carried out by 155 primary school teachers has revealed a "shocking lack of mental arithmetic ability and basic maths knowledge".

Fewer than four out of 10 of those who sat the test – designed for 11-year-olds – could calculate 2.1% of 400, and only a third answered correctly that 1.4 divided by 0.1 was 14. Overall, four out of ten scored 40% or below, only one got all the answers correct and the average mark was 12 out of 27 or 45%.

Alison Wolf, professor of public sector management at King's College London, said: "I am horrified by the statistics. I think that our obsession with generic teaching skills has crowded out time in which we could be making sure that people have the basic content and knowledge of content that they need."

The test, carried out for the Channel 4 documentary series *Dispatches*, included addition and multiplication sums, simple algebra and questions involving fractions, conversions and averages. Teachers performed well on some of the easier questions. For example, 97% were able to work out $2 \times 5 - 4 = 6$ and 75% knew that three sevenths of 21

Today

- ▣ Let's show some more examples with Nspire-CAS software, where we motivate math topics using technology, showing students that they have to know all the basic rules to understand the results we get using the calculator or the software and showing also how to ask the right questions...
- ▣ We also have to consider where are we going with technology for doing math. For example, take a look at Wolfram Alpha website (there is also an app available): <http://www.wolframalpha.com/>
- ▣ Just enter a simple expression, for example $x^2 + 15x + 56$
- ▣ Not knowing what you want, the system will give you everything you could possibly want with this (graph, roots, factorisation etc.). It's like when Google completes your question for you...

CONCLUSION

- ▣ After 15 years of mandatory use of CAS calculators, we can say it has had a big impact on teaching to our engineering students
- ▣ We think we should still be teaching all the basic topics and techniques, using technology to show how these apply
- ▣ We should be using technology to motivate presentation of even more mathematical concepts
- ▣ We will be seeing less paper pencil calculations for symbolic calculations, the CAS system will be more and more efficient, being able to do all calculations usually done by hand.

CONCLUSION

- ▣ In 20 years, what will math education be like?
- ▣ To answer, we should also ask what will technology look like...
- ▣ We should expect all basic algebra and calculus operations being done with technology
- ▣ More than ever, we will need to teach mathematics, the language, rules...
- ▣ Tablets? Handwriting math recognition, vocal math commands... For sure!
- ▣ Thank you!